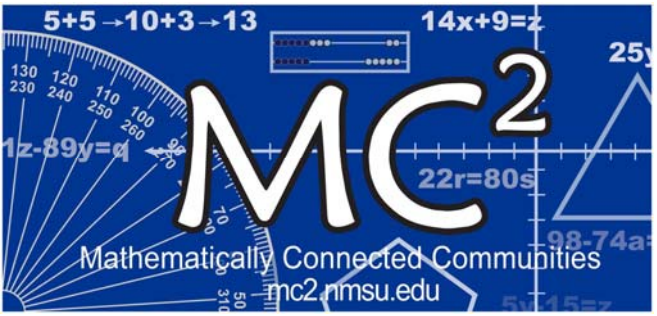


Welcome!

This webinar was pre-recorded on Wednesday, January 27, 2016.

Please submit any questions or comments to mc2@nmsu.edu.

From Counting to Counting On

Fostering the Development of Early Addition & Subtraction

Narrators: Megan Kidwell and Lisa Matthews
Developers: MC^2 K-3 Team

Stages of Early Counting:
Research

- Les Steffe, UGA, 1980s
- Identifies stages of counting that children progress through as they come to understand the operations of addition and subtraction
- Related to Piaget's theory of cognitive development

Stages of Early Counting:
Making Sense with Concrete Collections

- Emergent
  Student is unable to accurately count a collection

- Perceptual
  Student can count visible collections, but not collections that are covered up

- Figurative
  Student can count covered collections, but always "counts everything" (always starts counting from one)

- Counting On /Counting Back
  Student is able to start with one collection and count on or back

Stages of Early Counting:

Making Sense with Concrete Collections

- Emergent
  Student is unable to accurately cou
- Perceptual
  Stud at are
- S , but always "counts
  e unting from one)
- Co ng On /Counting Back
  Student is able to start with one collection and count on or back

ADVANCING TO EACH NEW STAGE INVOLVES A COGNITIVE REORGANIZATION REGARDING HOW COLLECTIONS WORK

Features of Early Counting

- Number sequences (verbal)
- One-to-one correspondence
- Keeping track
- One total for two distinct collections
- Cardinality:
  The answer to “How many?” can be a single number word (“5”)
- Progresses from concrete to abstract:
  visible collections → concealed collections

Features of Early Counting

- ☑ Knows number sequences (verbal)
- ☑ Uses one-to-one correspondence
- ☑ Monitors items as they are counted
- ☑ Can give the total for two distinct collections
- ☑ Cardinality:
  The answer to "How many?" can be a single number word ("5")

Perceptual Counting:
Students are counting things they can see

- ☐ Progresses from concrete to abstract:
  visible collections → concealed collections

Features of Early Counting

- ☑ Knows number sequences (verbal)
- ☑ Uses one-to-o
- ☑ [illegible]
- ☑ [illegible]
- ☑ C
  Th[illegible] to "How many?" can be a single number word ("5")

EMERGENT → PERCEPTUAL COGNITIVE REORGANIZATION:
ATTENDS TO CERTAIN FEATURES WHEN
DETERMINING QUANTITY

tual
ng:
unting
see

- ☐ Progresses from concrete to abstract:
  visible collections → concealed collections

Features of Early Counting

- ☑ Number sequences (verbal)
- ☑ One-to-one correspondence
- ☑ Keeping track
- ☑ One total for t… ections
- ☑ …be a single number word (“5”)

Figurative Counting:
Students are counting things that are covered up

- ☑ Progresses from concrete to abstract:
  visible collections → concealed collections

Features of Early Counting

- Number sequences (verbal)
- One-to-one correspondence
- Keeping track
- One total fa
- Fi
- ord ("5")
- act:
- concealed collections

PERCEPTUAL → FIGURATIVE COGNITIVE REORGANIZATION:
DOES NOT HAVE TO BE ABLE TO SEE OBJECTS IN ORDER TO COUNT THEM

Features of Early Counting

- ☑ Counting on / counting back

  Student understands that, rather than counting everything in the first collection, they can refer to the entire first collection with a single number

  Student counts on to add, counts back to subtract

Features of Early Counting

- Counting on / cou
  - refer to the
    ngle number
  - ounts on to add, counts back to subtract

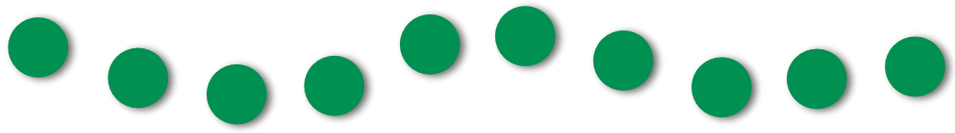
FIGURATIVE → COUNTING ON COGNITIVE REORGANIZATION:

A NUMBER CAN BE A COMPOSITE.

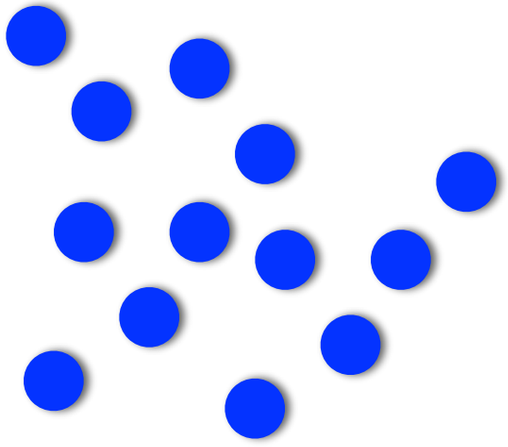
"6" INCLUDES ALL THE NUMBERS THAT COME BEFORE IT

Stages of Early Counting: Emergent Counting

Counting Collections



How many counters are there?



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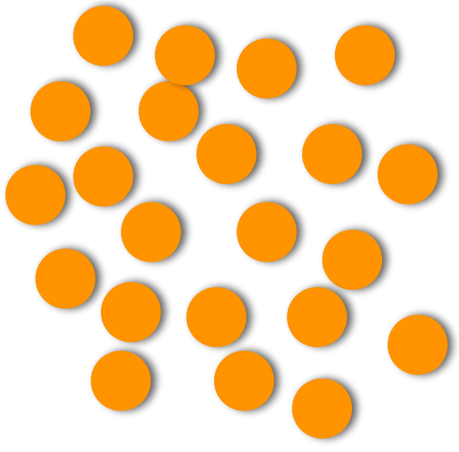
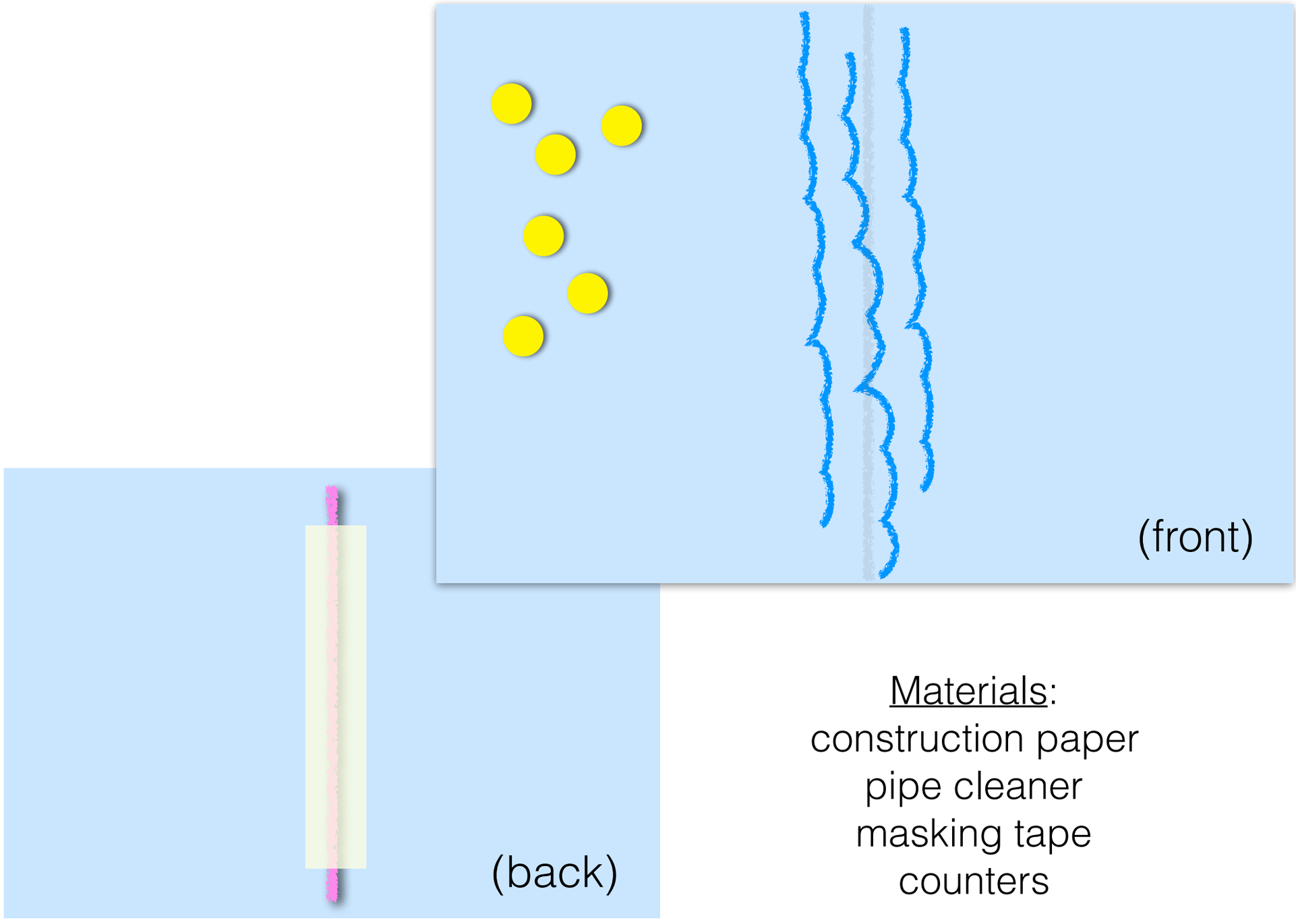


Can you get me 12 counters?

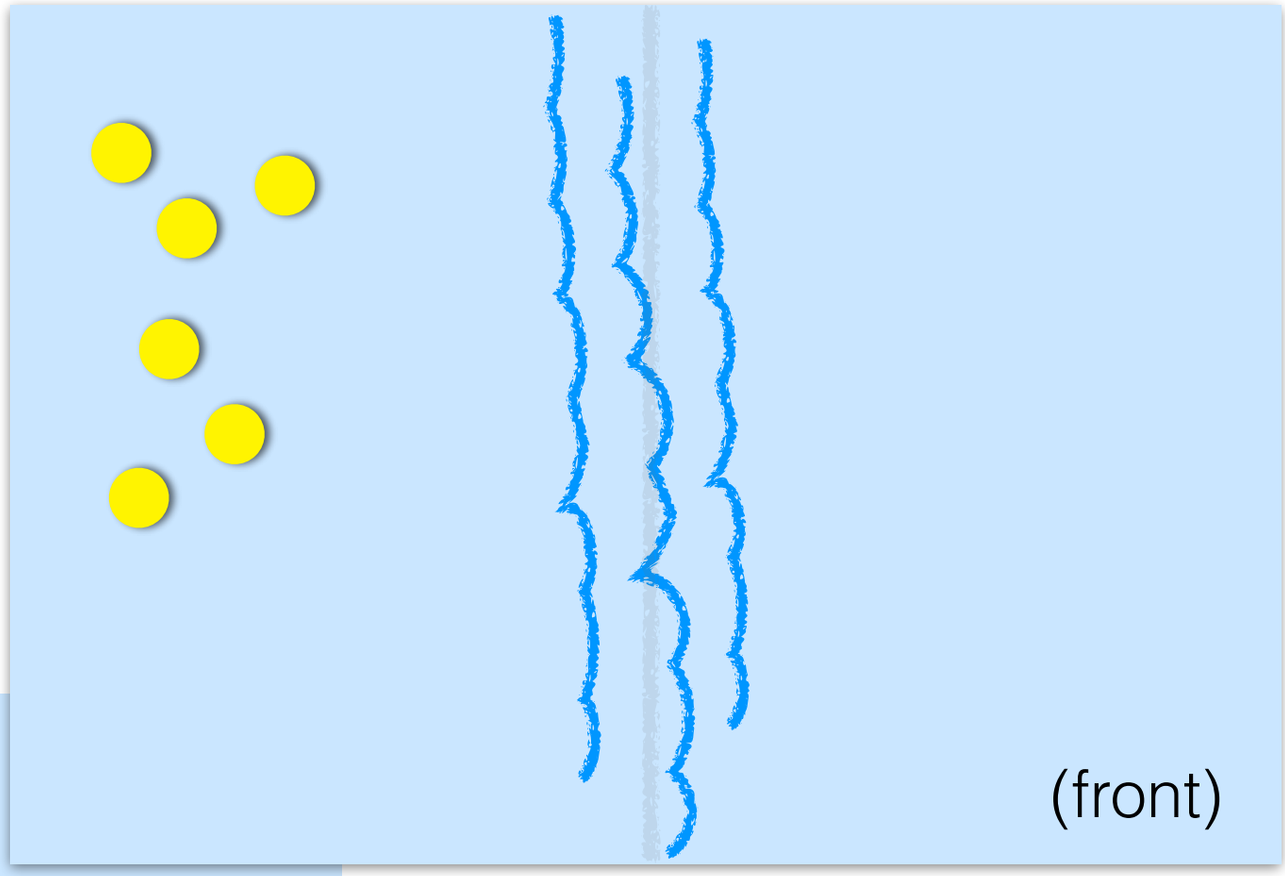
Counting Collections: Cross the River

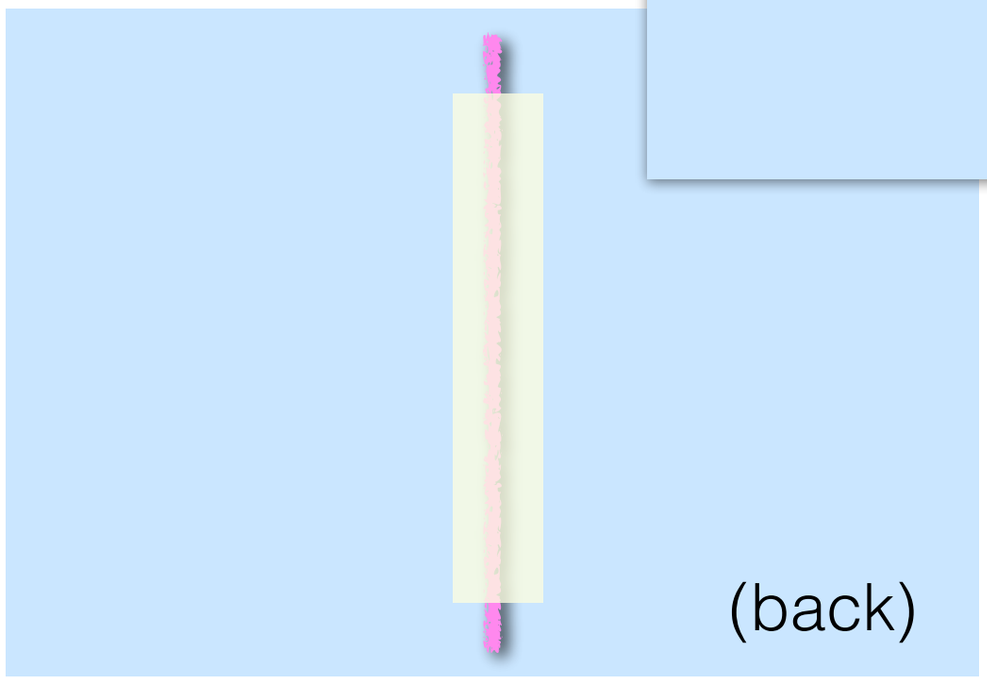


Materials:
construction paper
pipe cleaner
masking tape
counters

Counting Collections: Cross the River

These “ducks” need to cross the river.
Can you help them?
When you are done, I want you to tell me how many ducks crossed the river.





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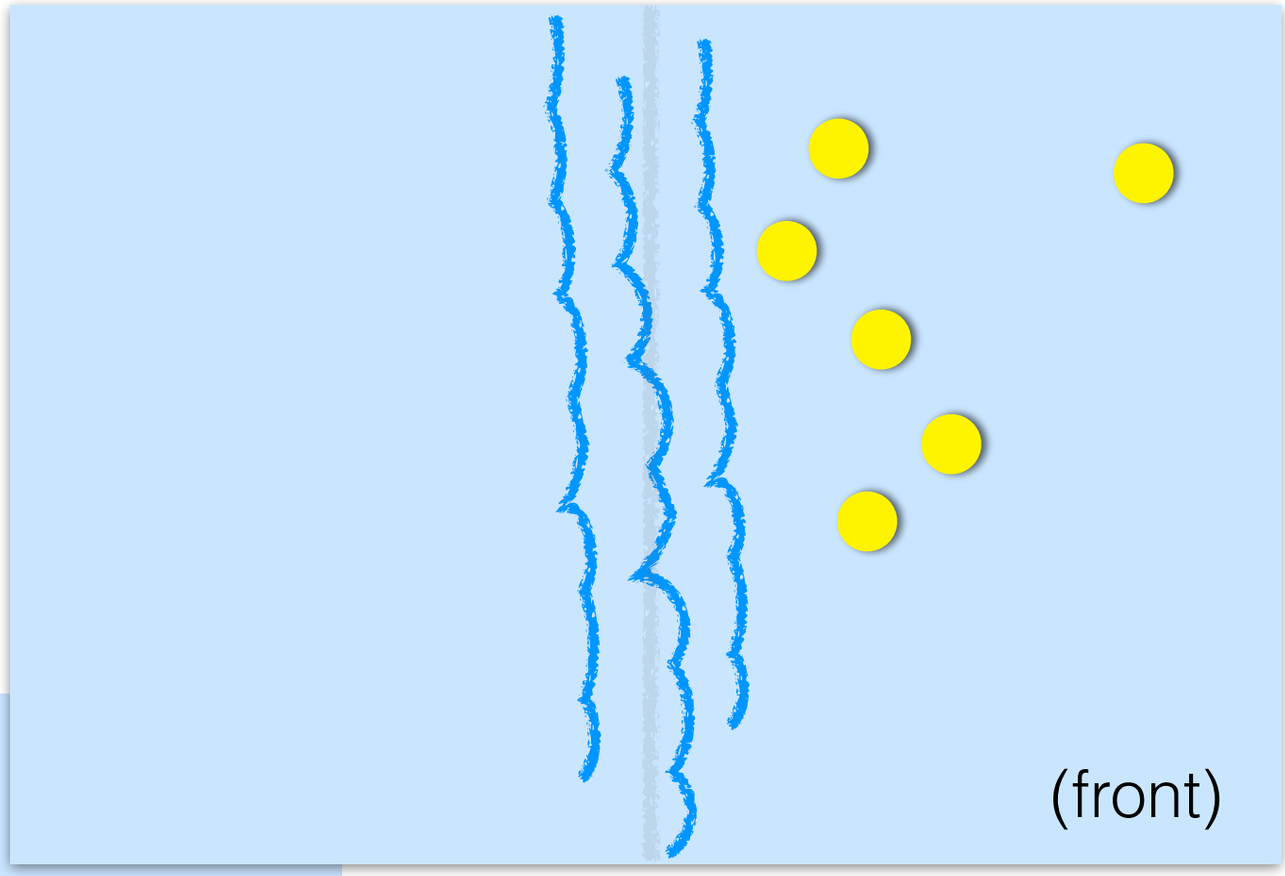

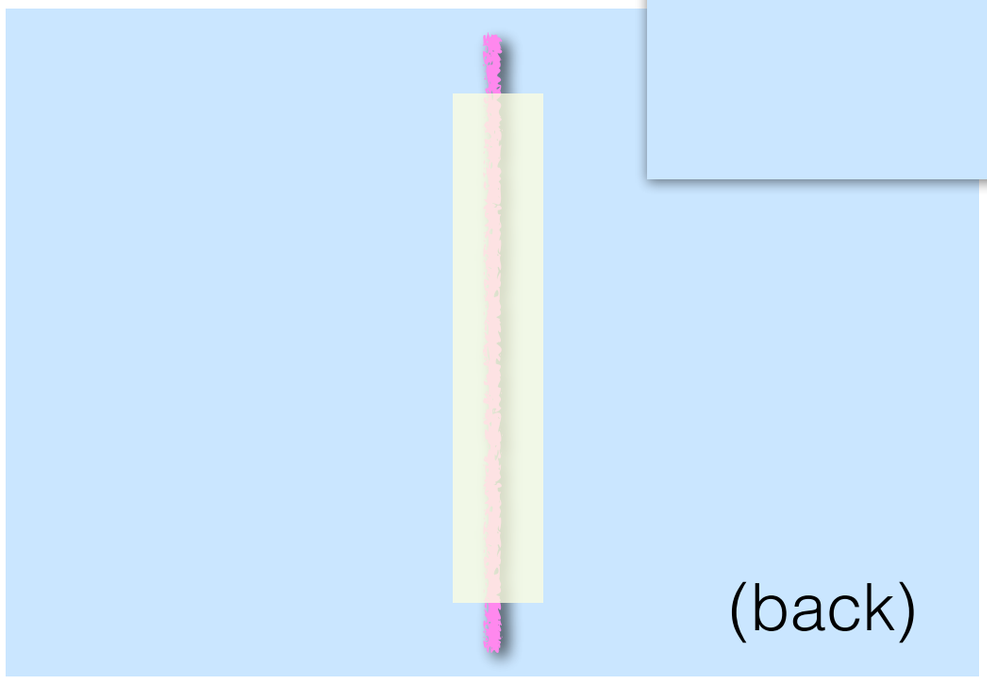

Materials:
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Number Row Count

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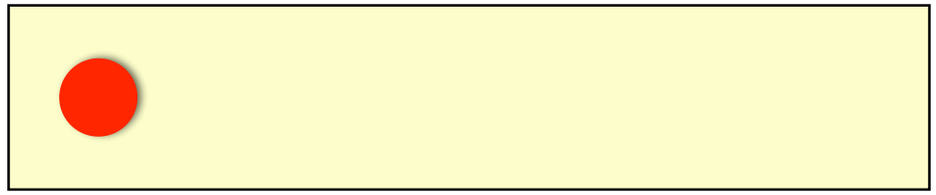

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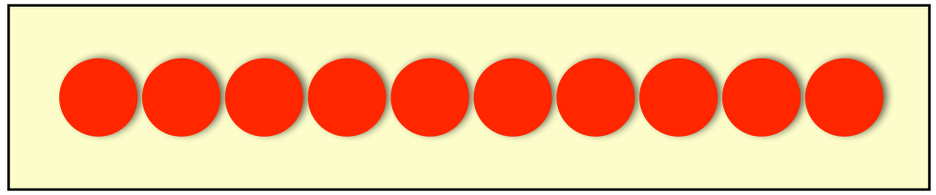

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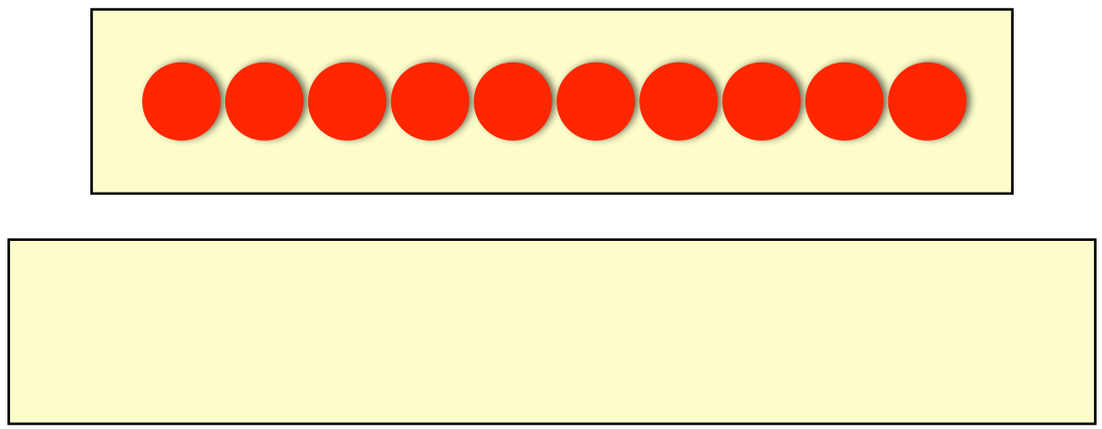

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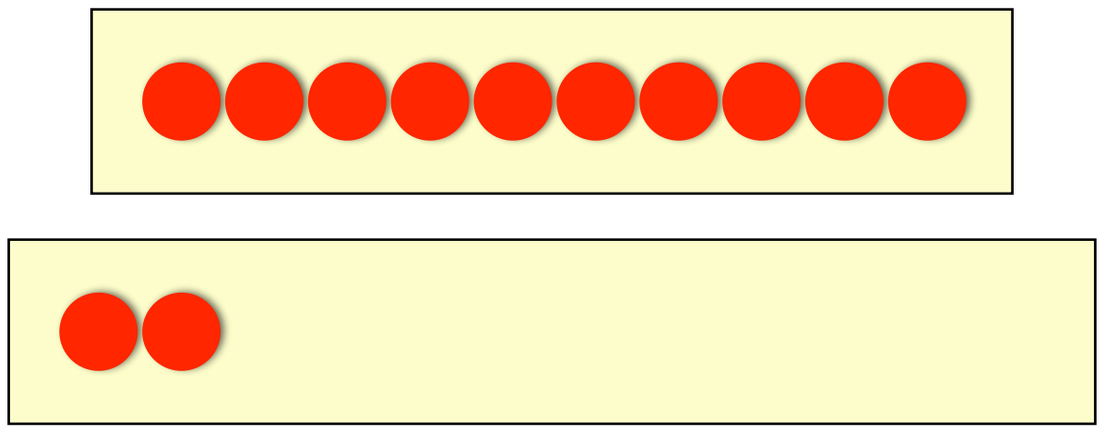

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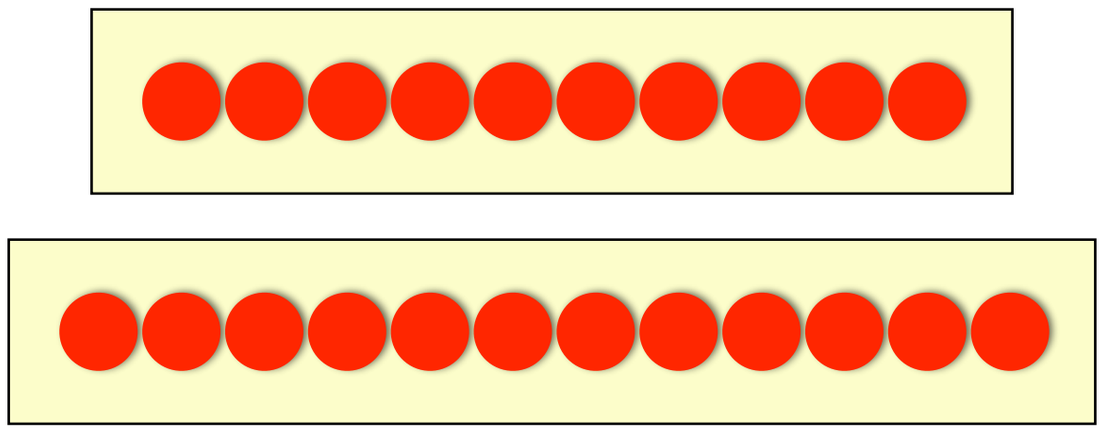

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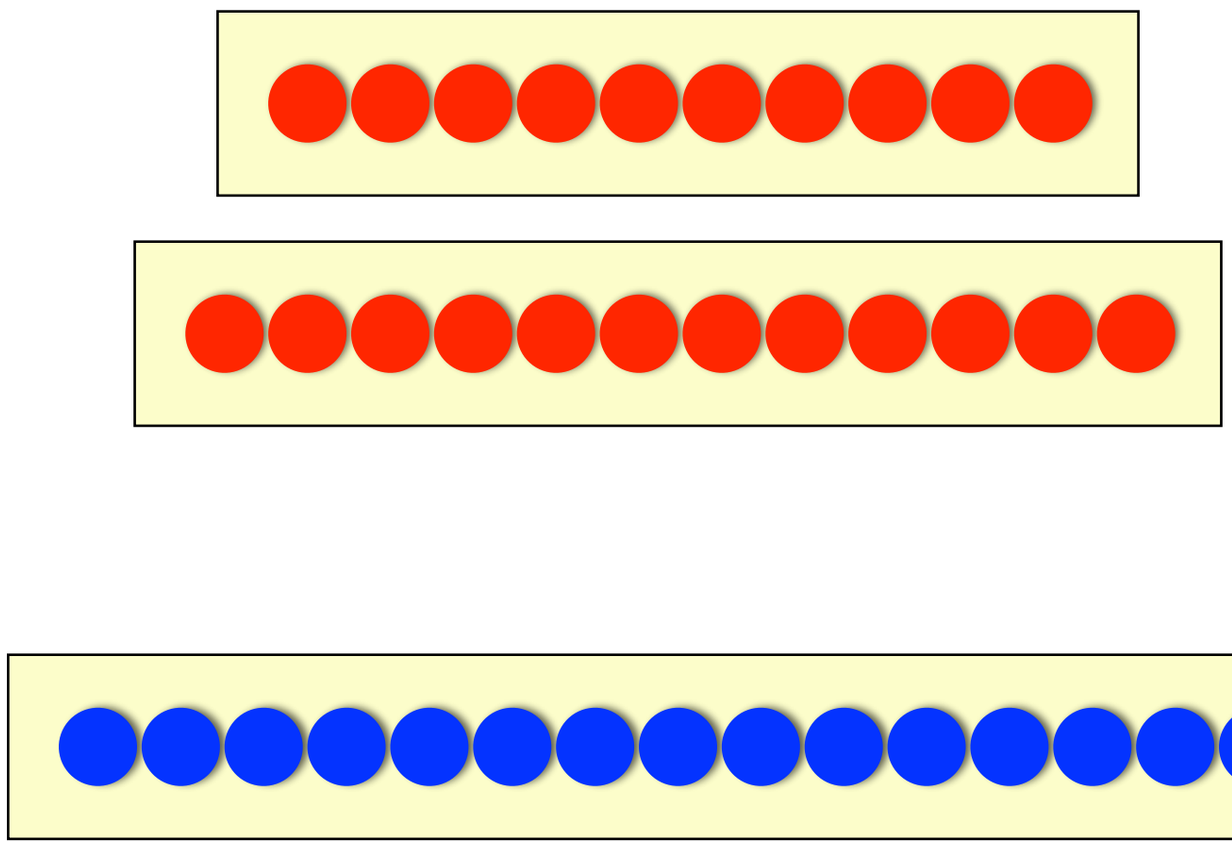

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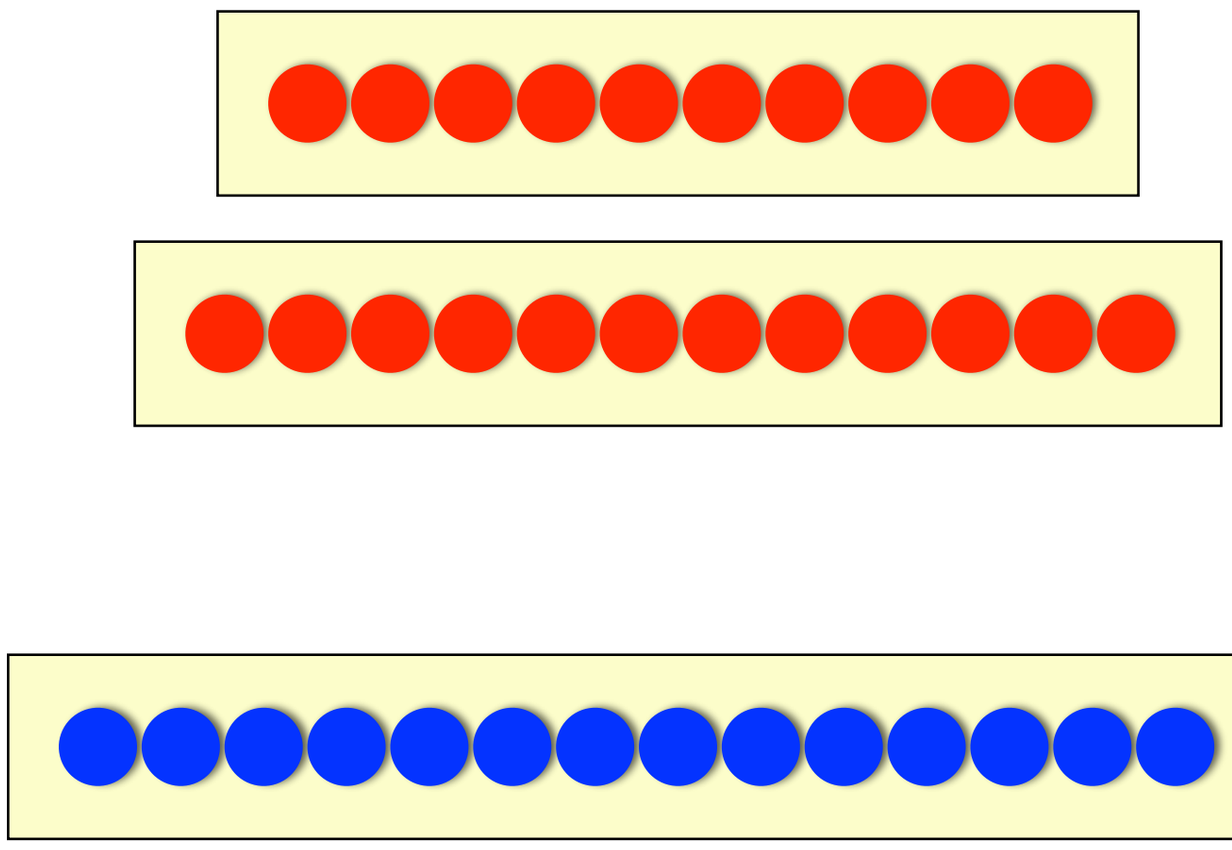
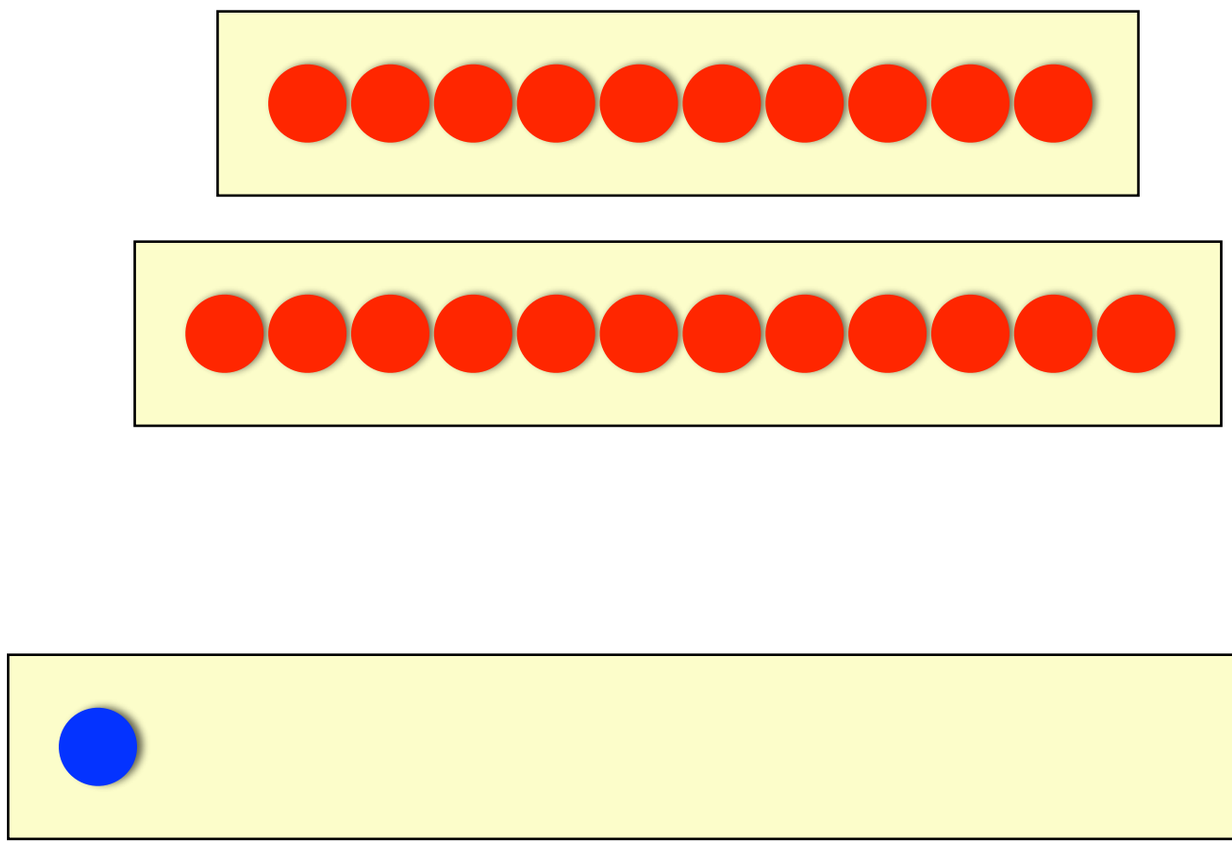


Number Row Count



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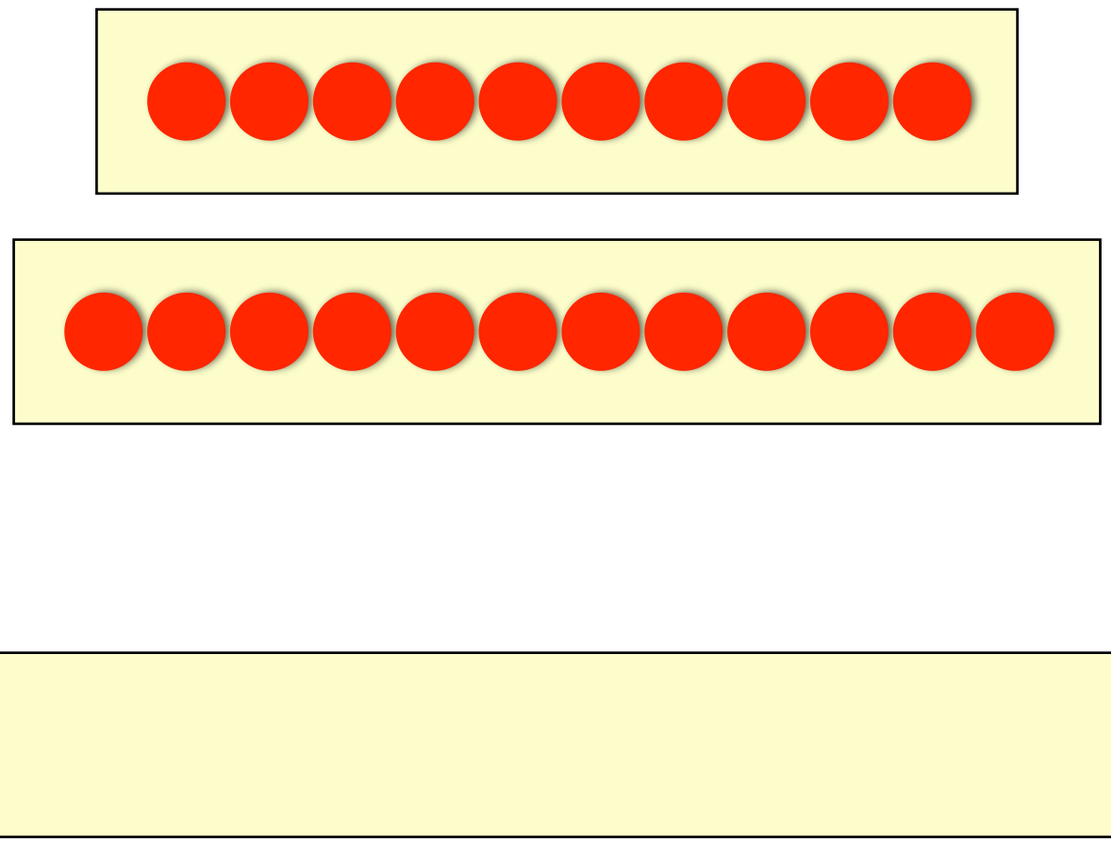

Stages of Early Counting: Perceptual Counting

- Number sequences (verbal)
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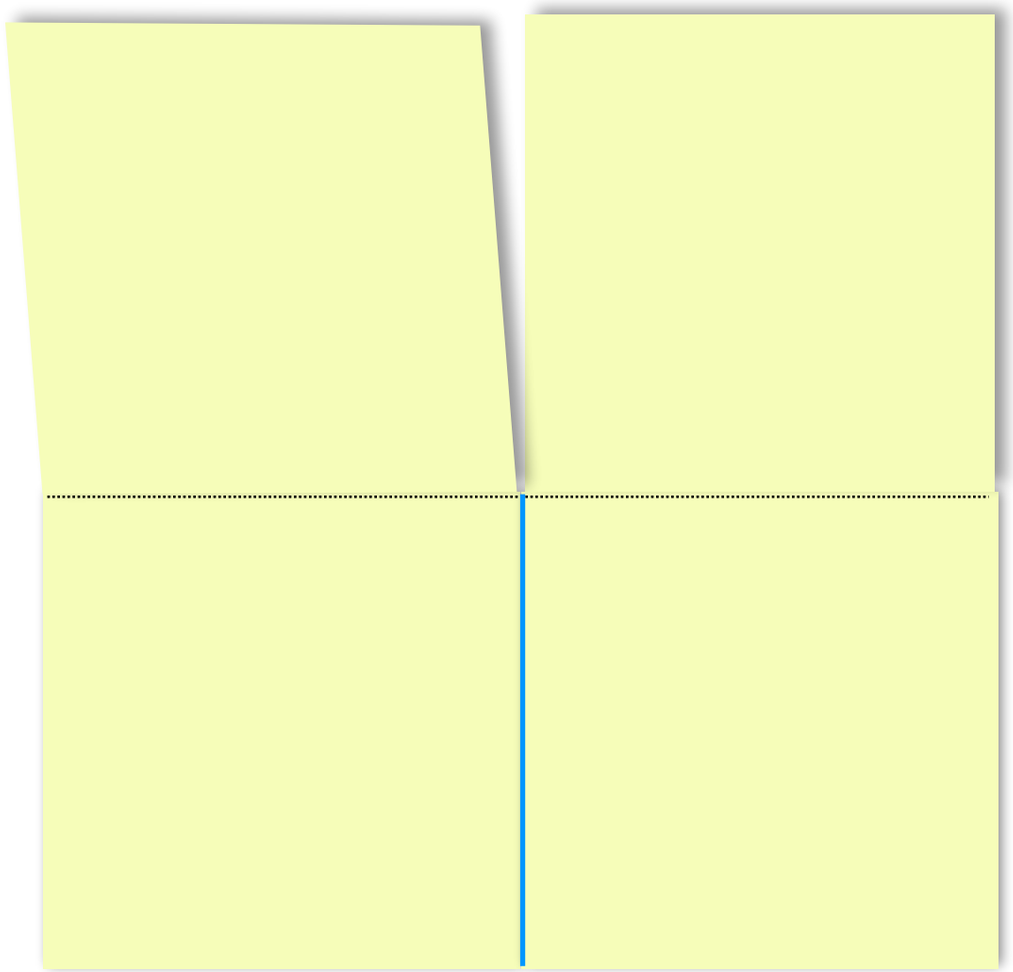
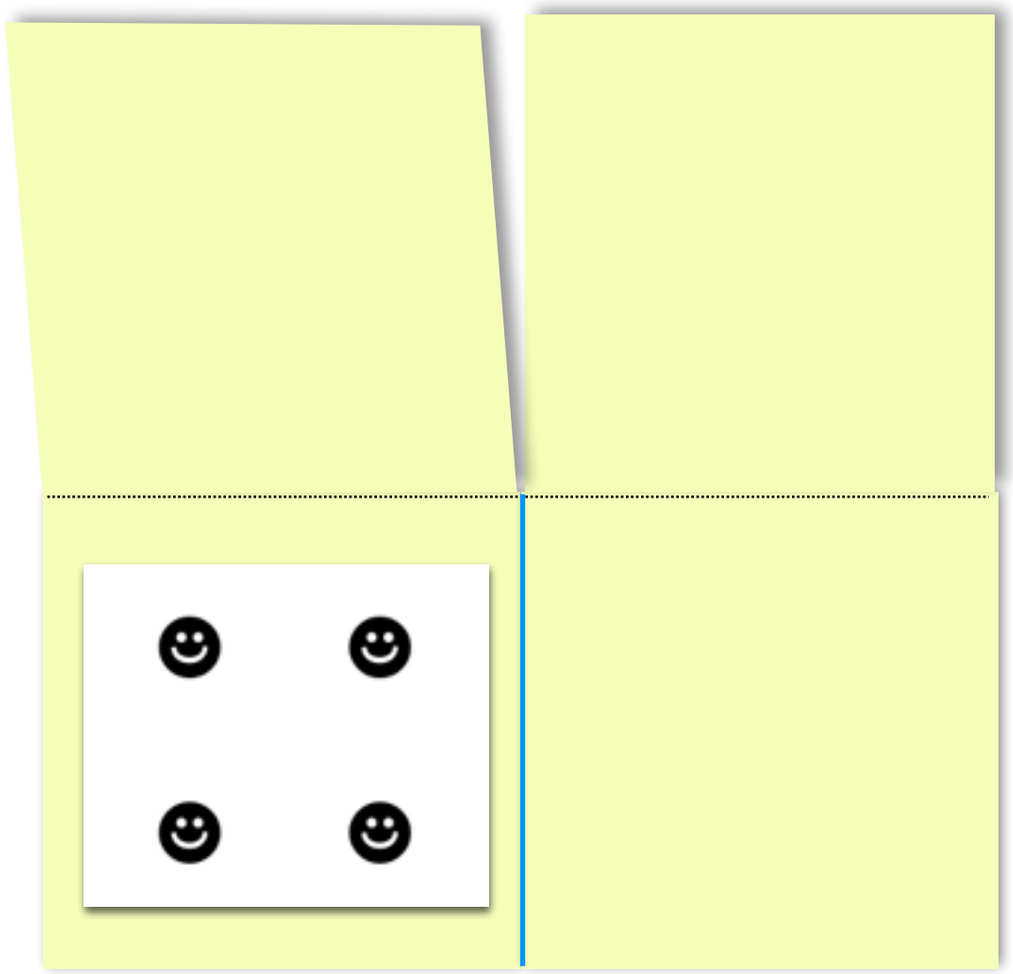
Domino Addition

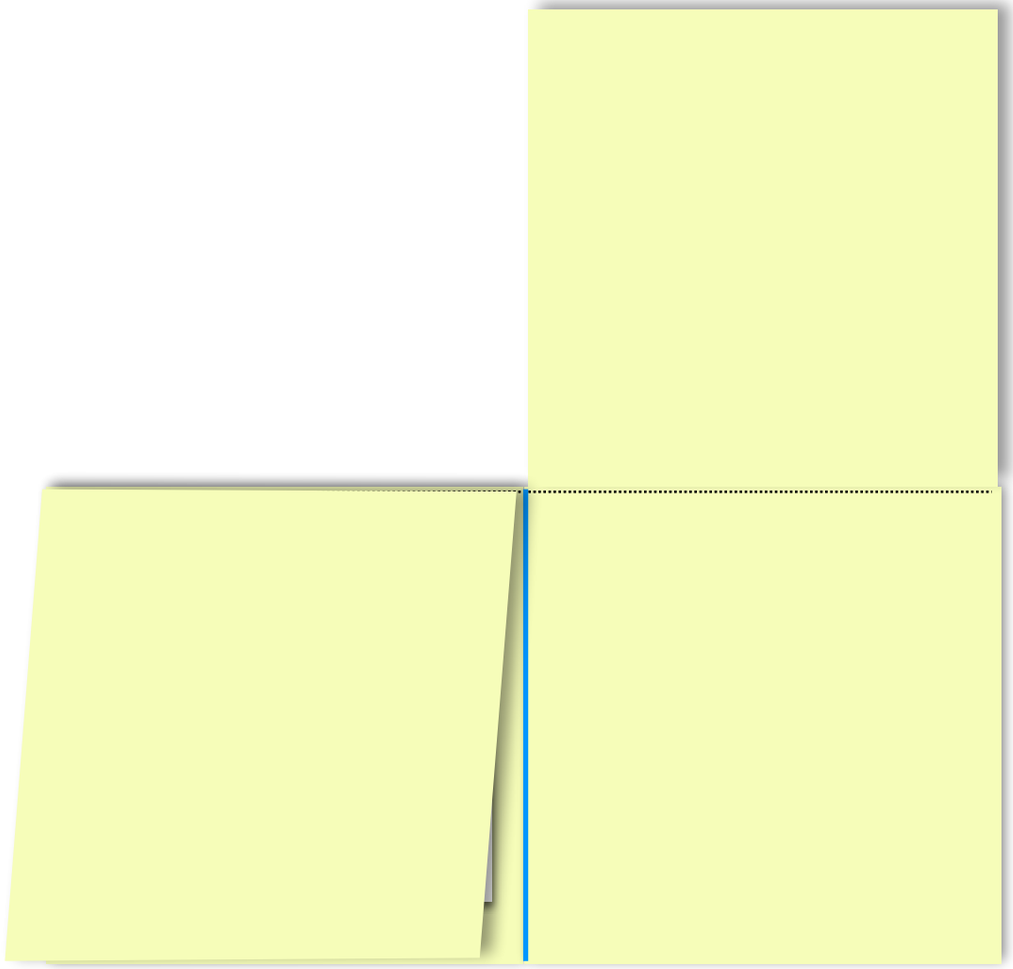


Domino Addition



How many dots do you see?

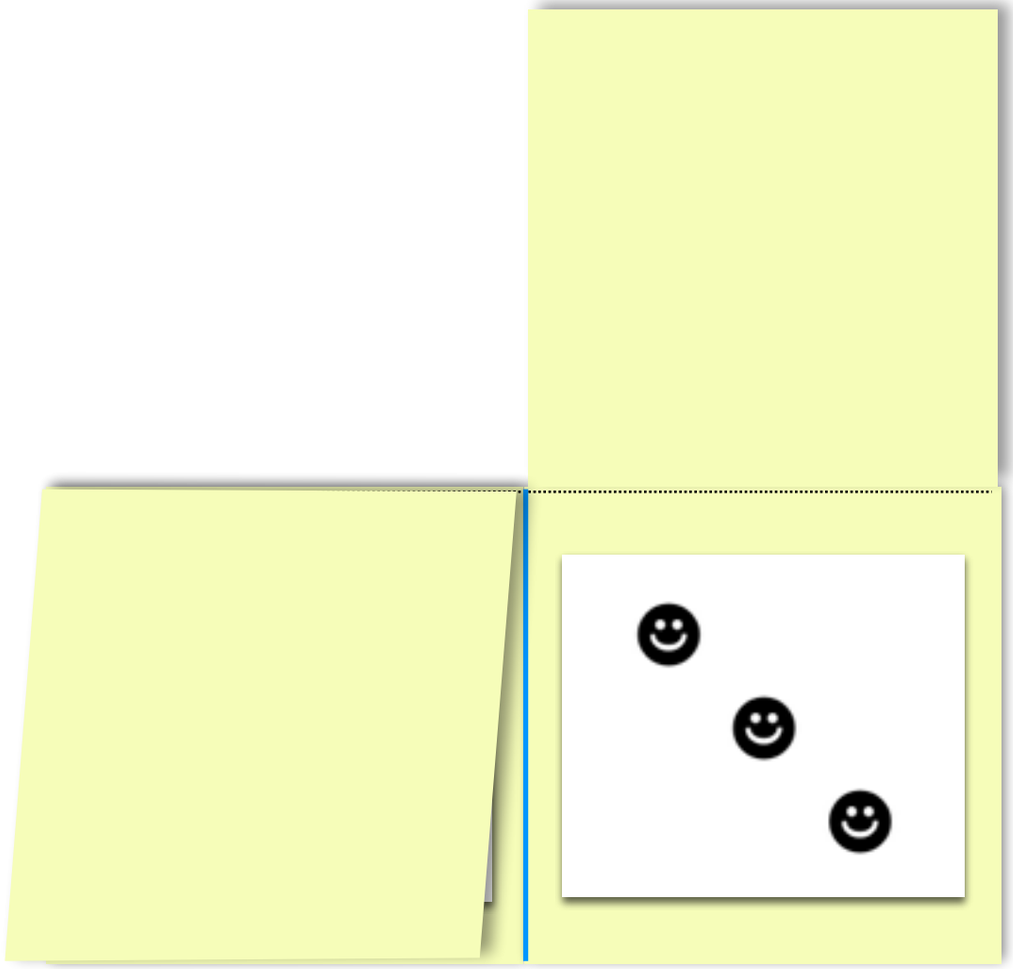
Domino Addition



How many dots do you see?

How many dots are under the door?

Domino Addition



How many dots do you see?

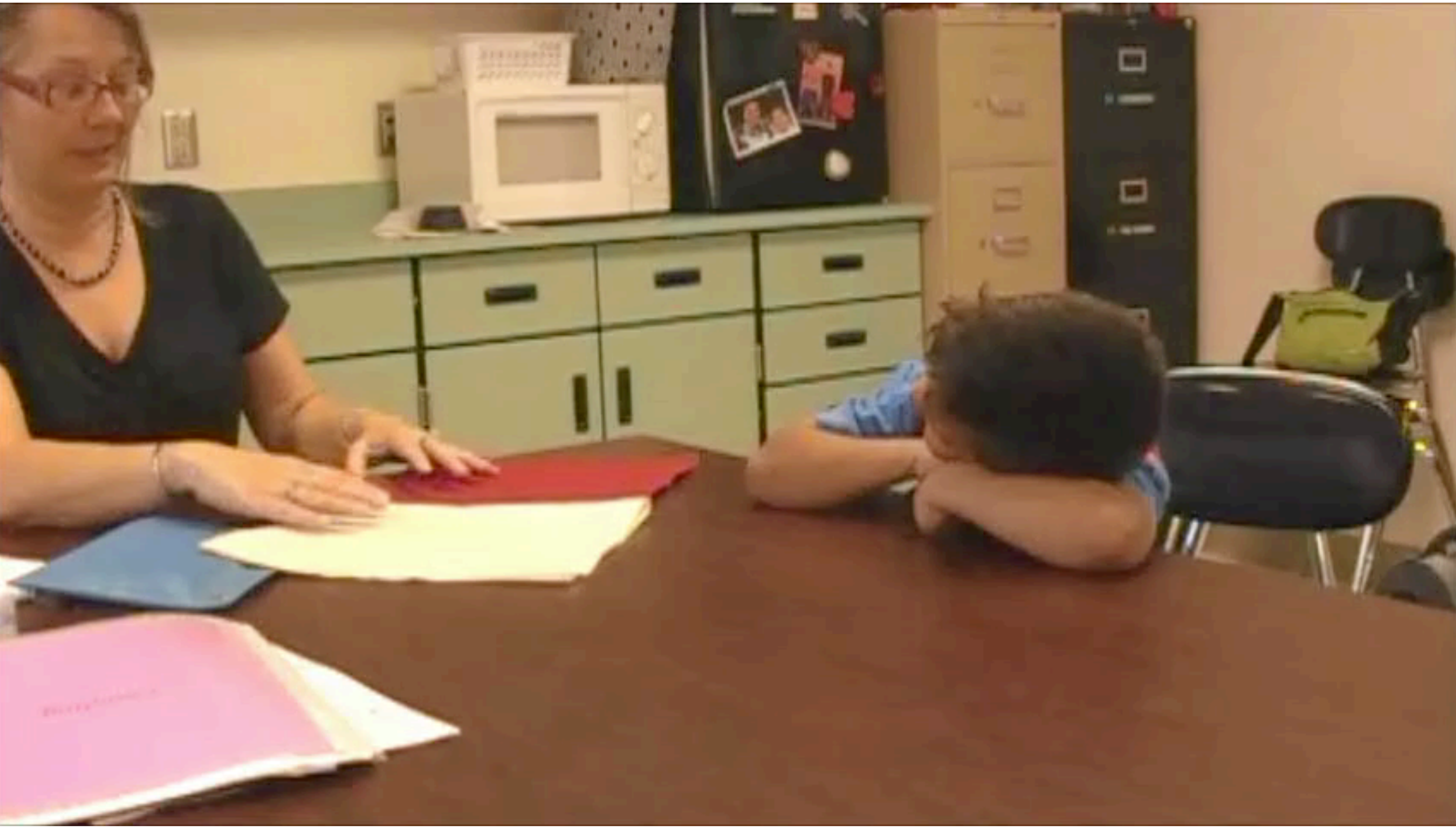
How many dots are under the door?

How many dots do we have all together?

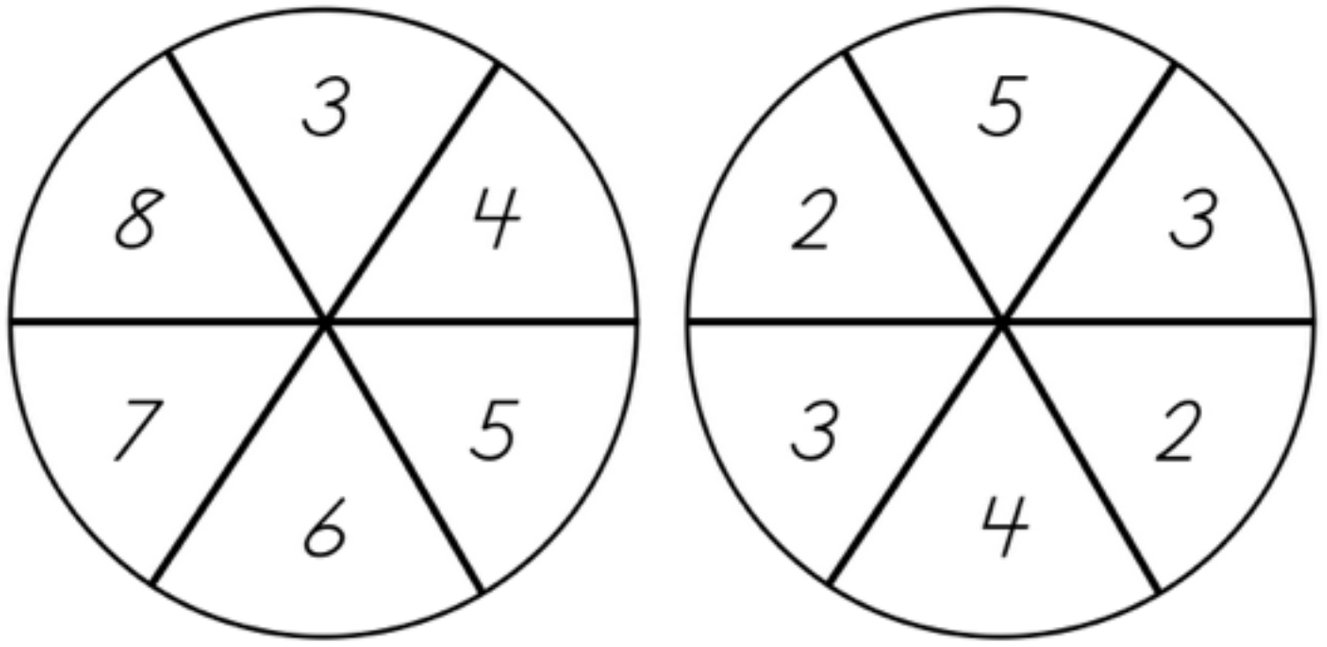
Variations: Conceal only the second card / Conceal both cards

Stages of Early Counting: Figurative Counting

- Progressing from concrete to abstract:
  - ~ counts collections that are covered up
  - ~ always counts from one (counts *everything*)



Addition Spinners



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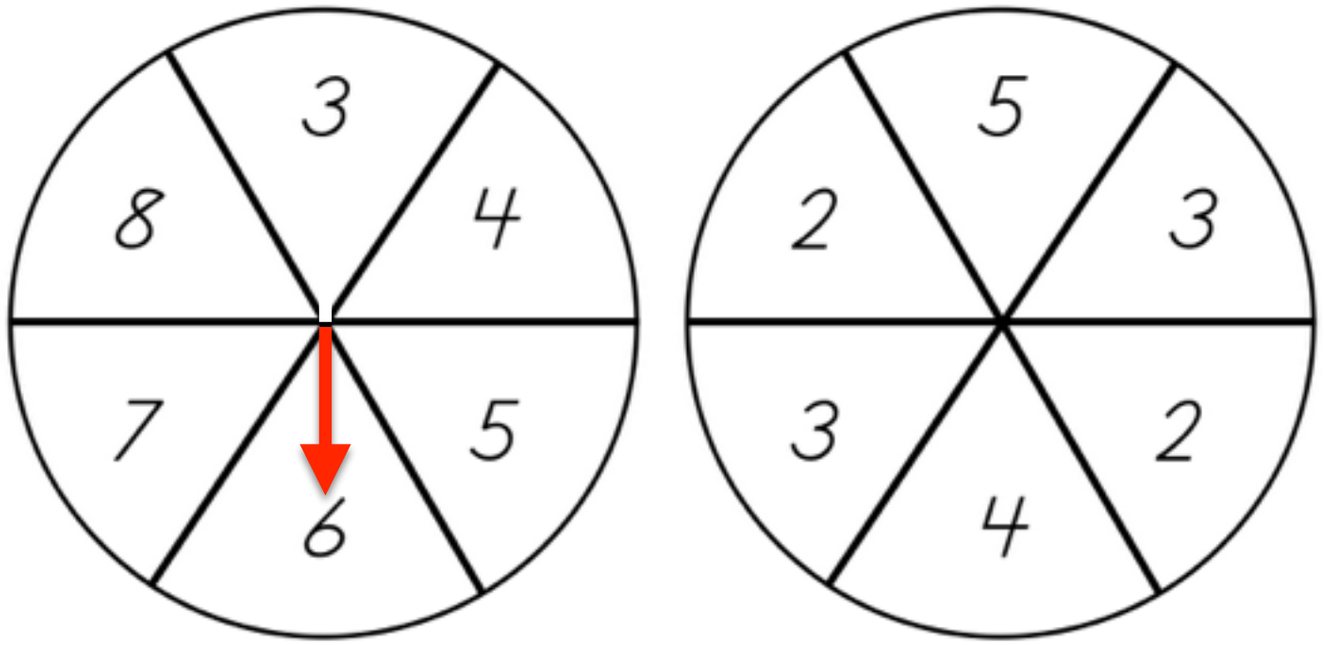

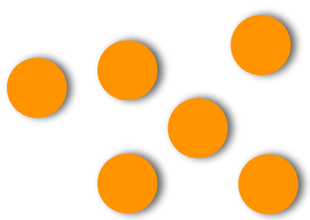

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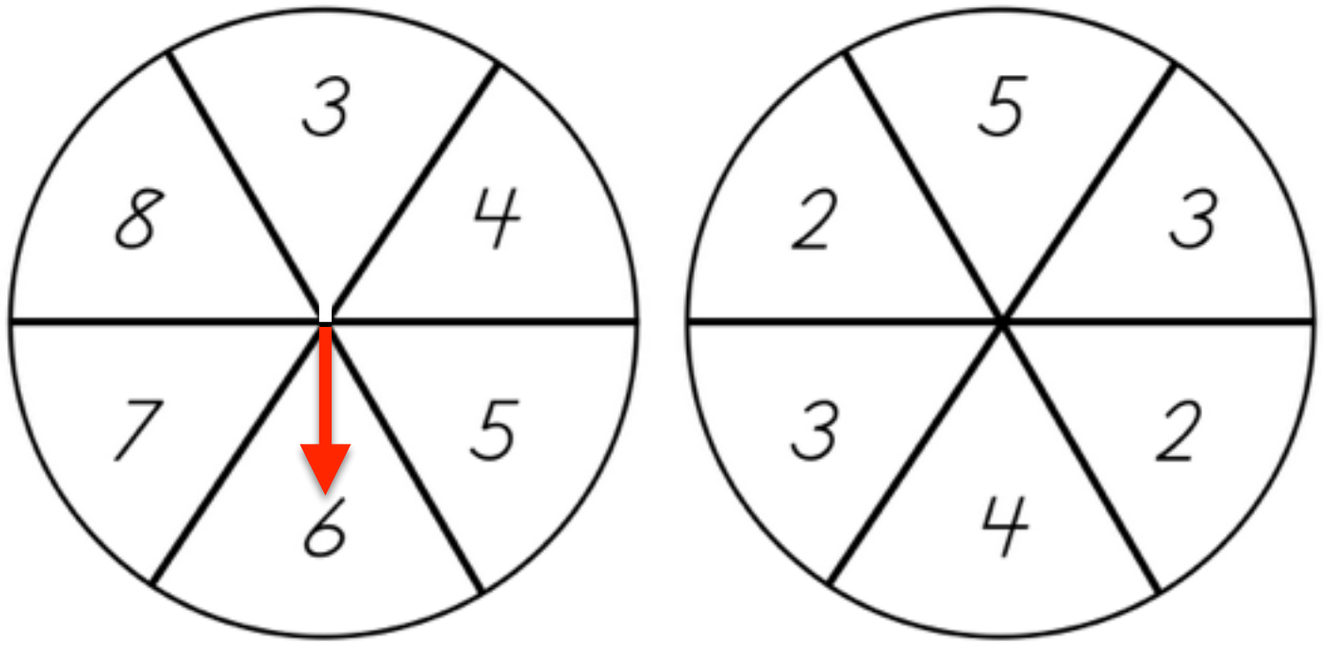

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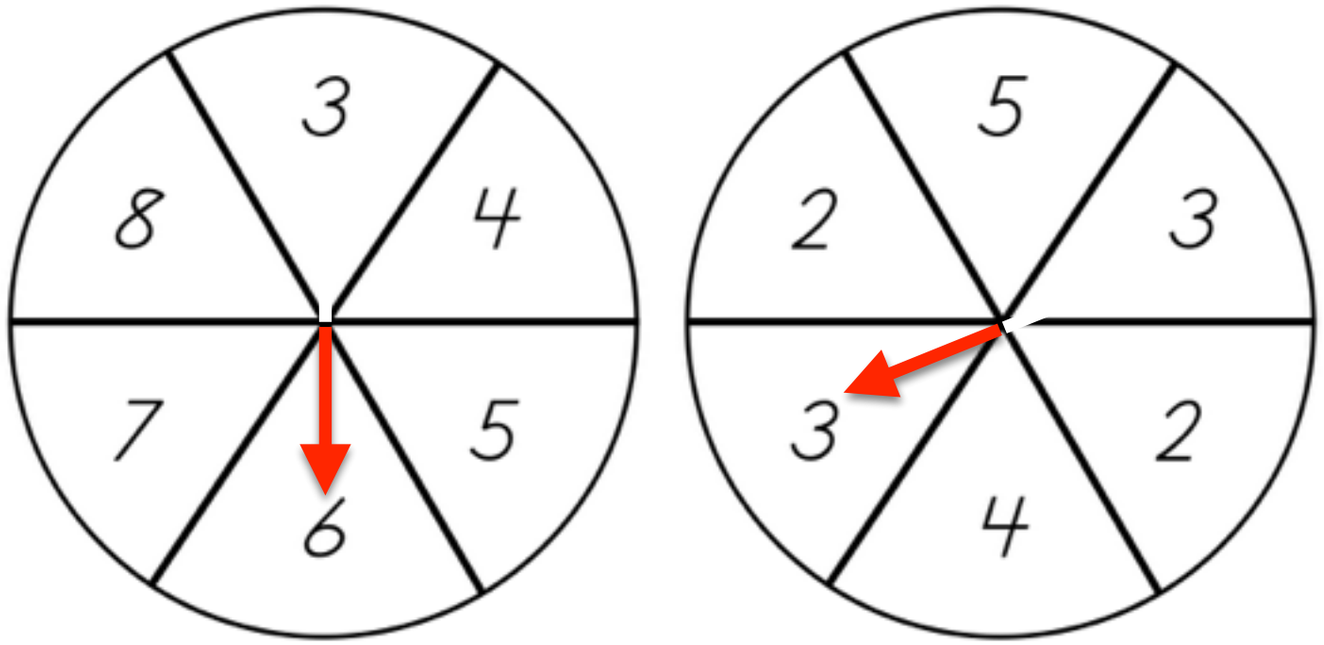
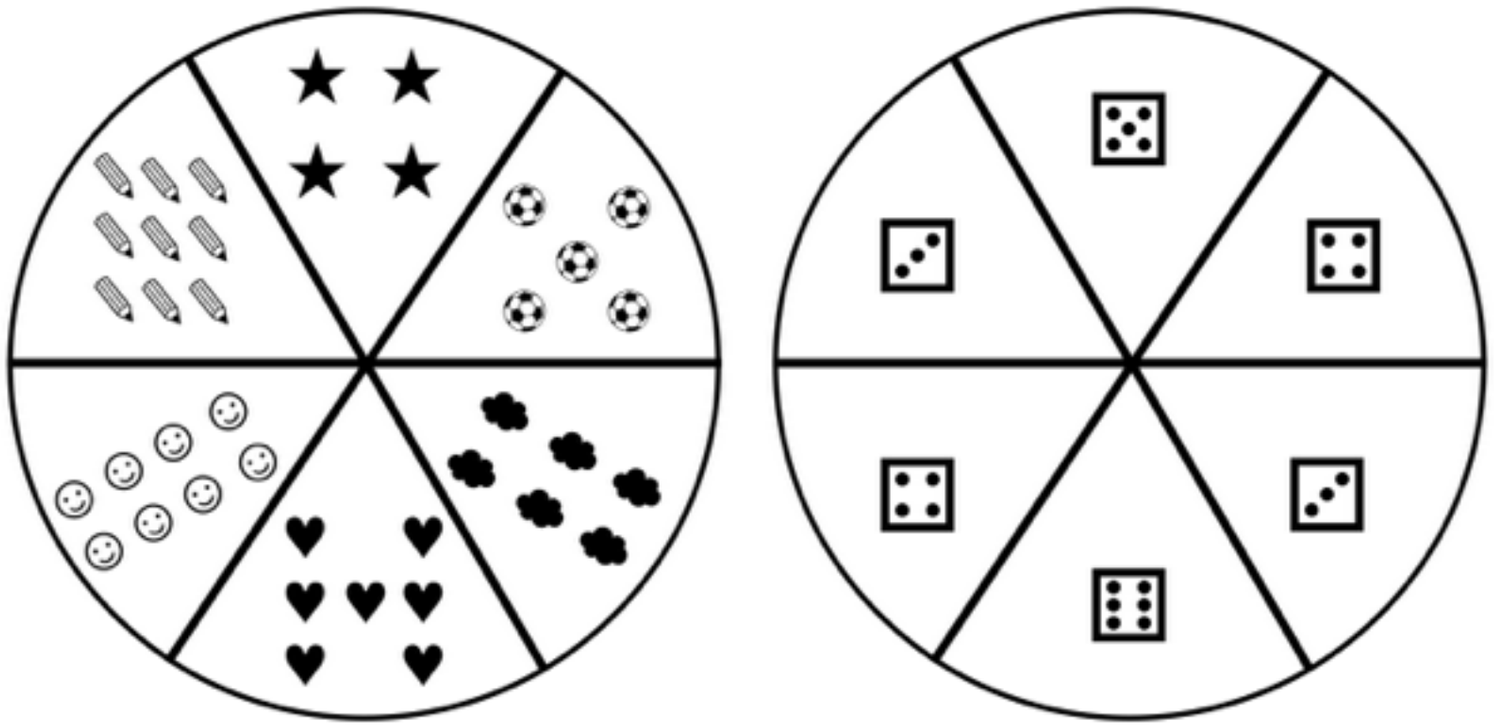


How many counters are there all together?

Hide & Add



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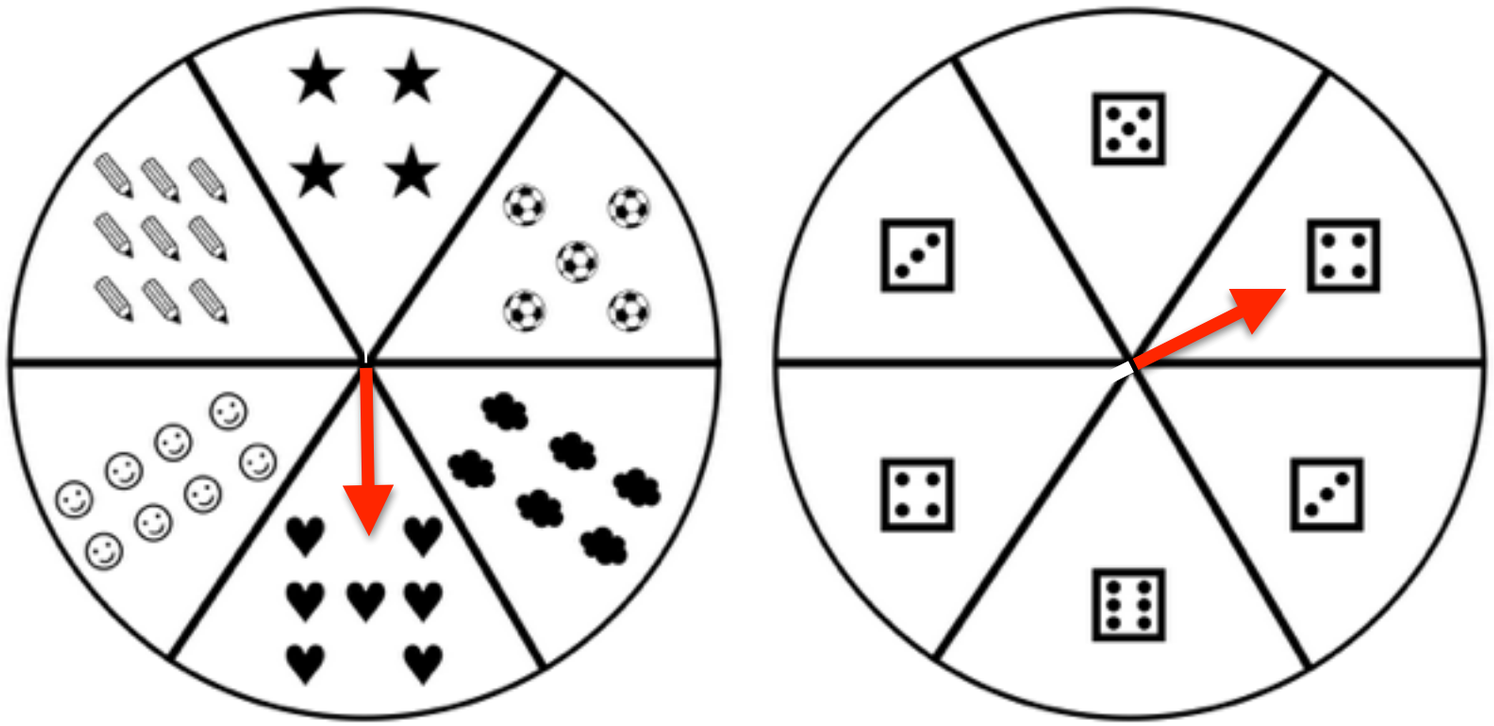

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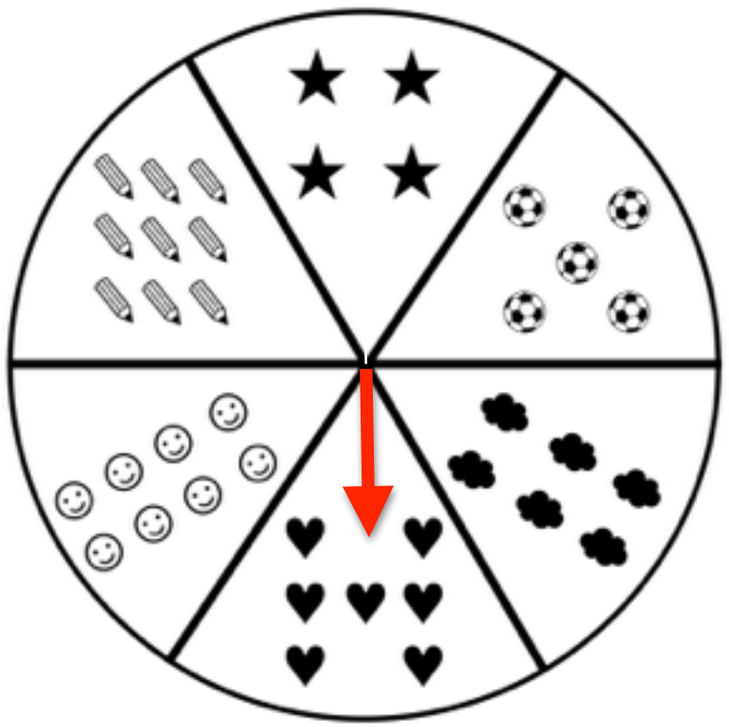
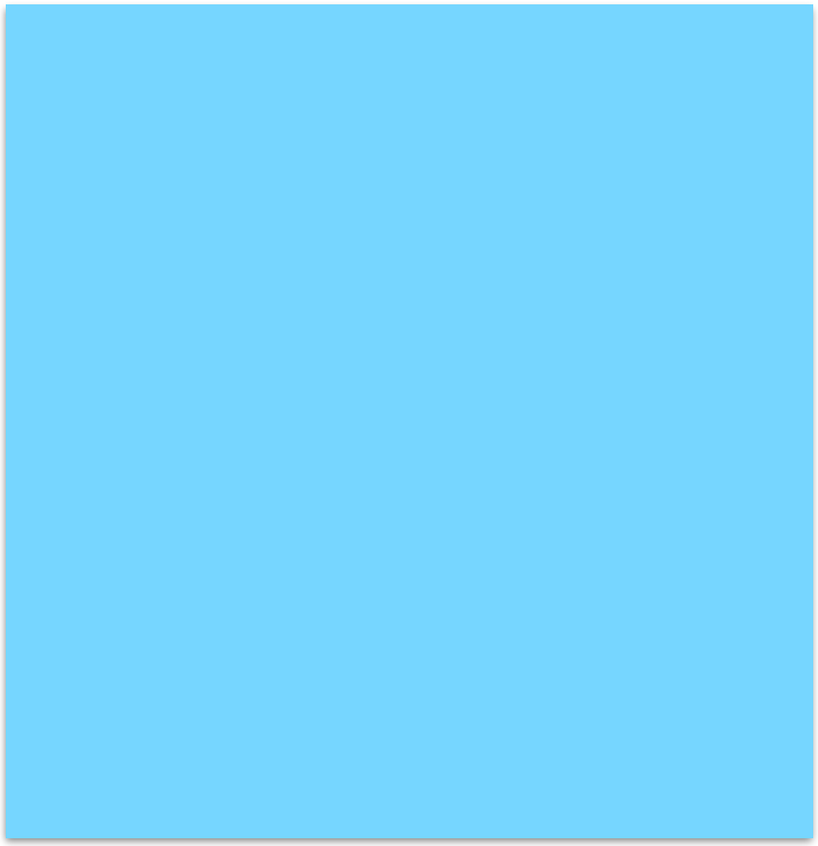
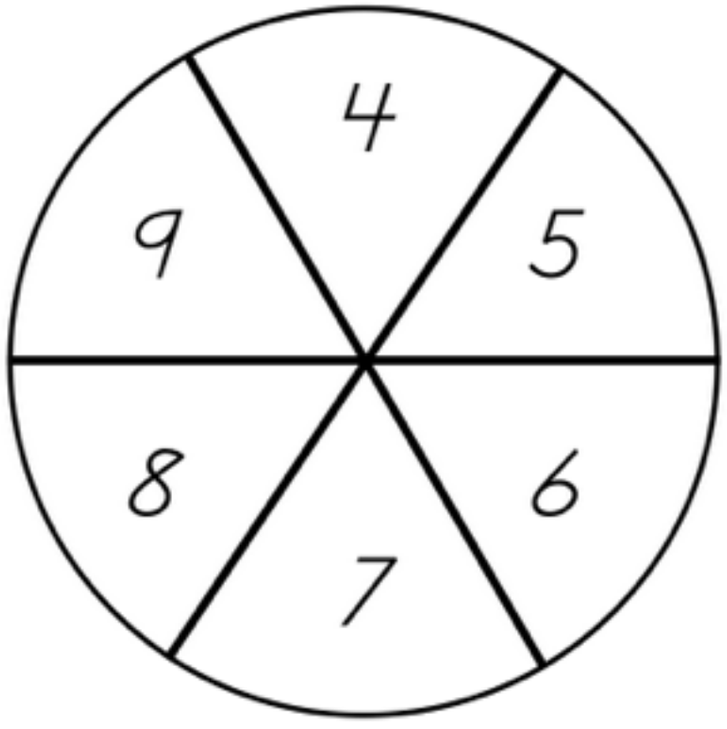
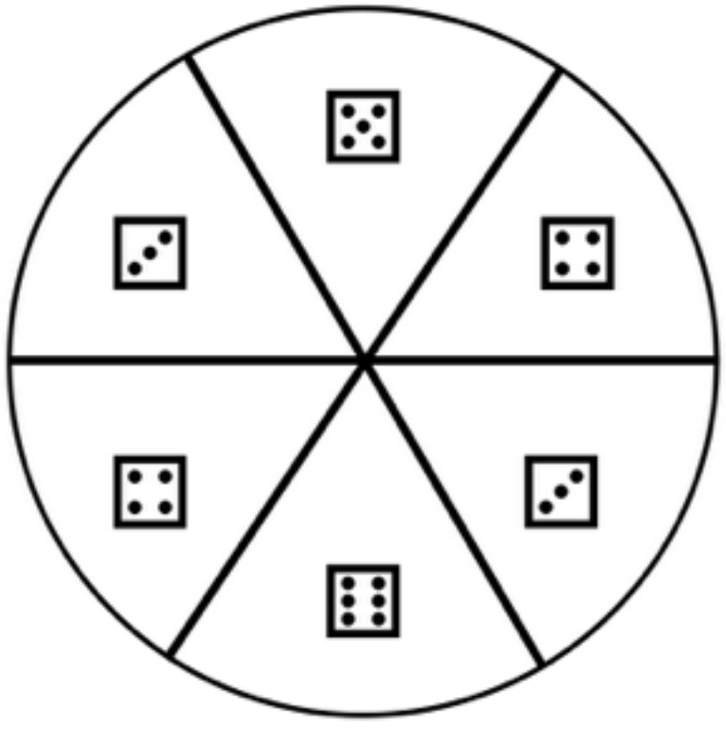




How much is 7 and 4 more?

Hide & Add





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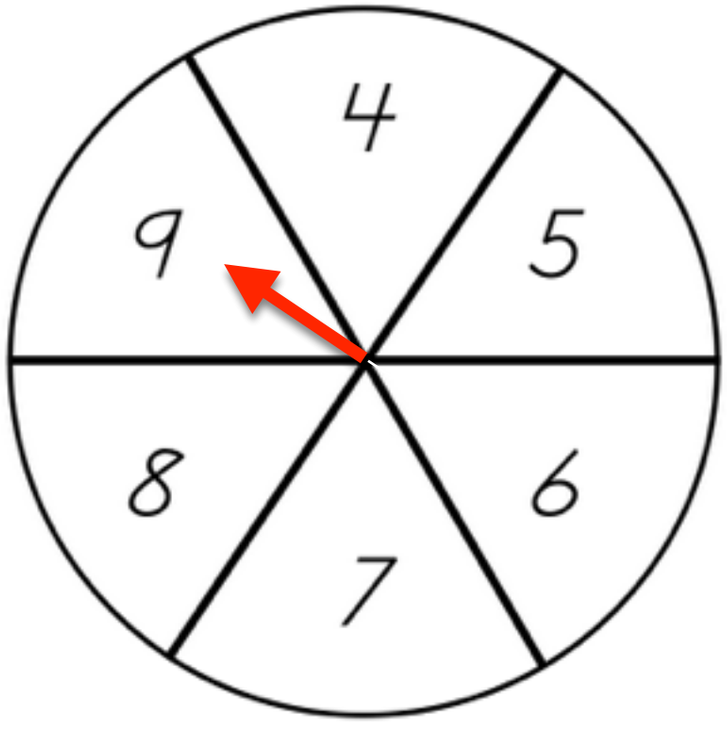

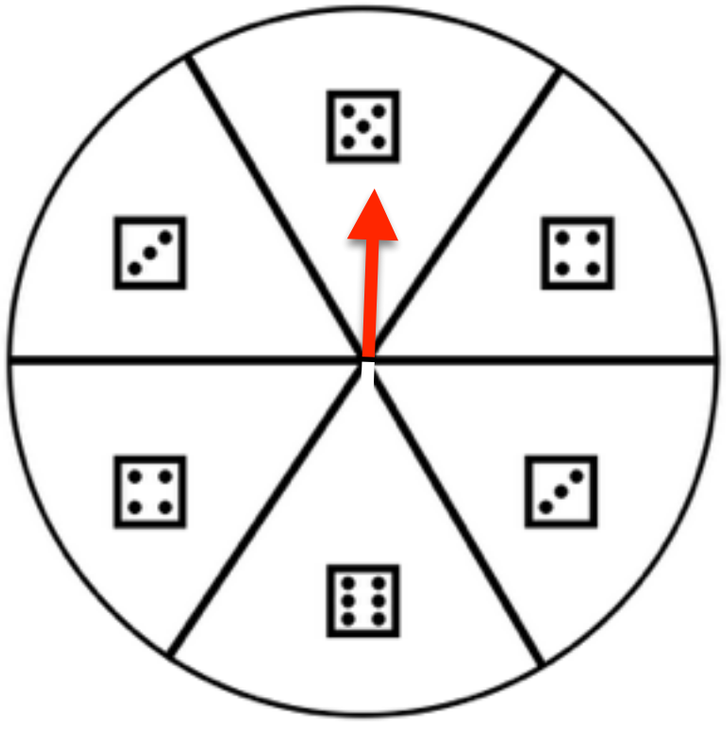

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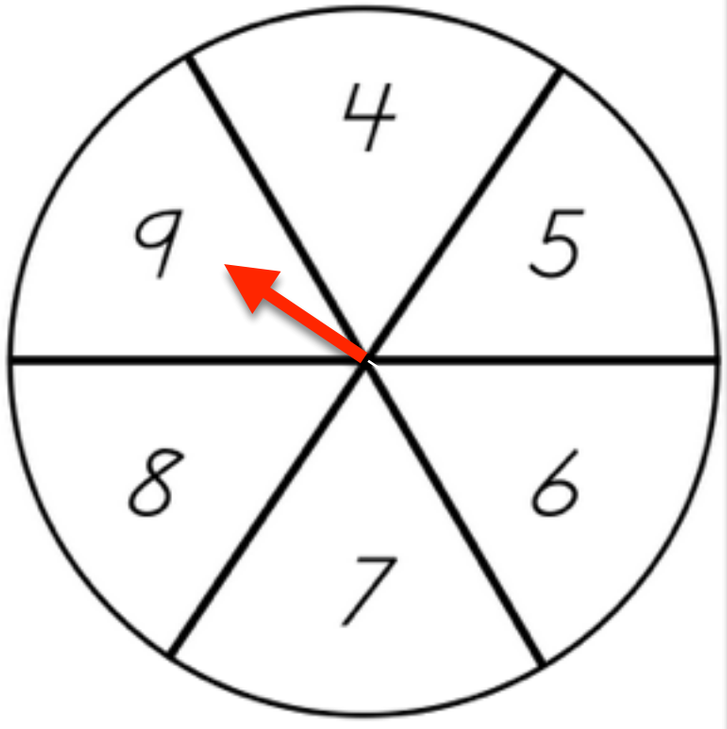
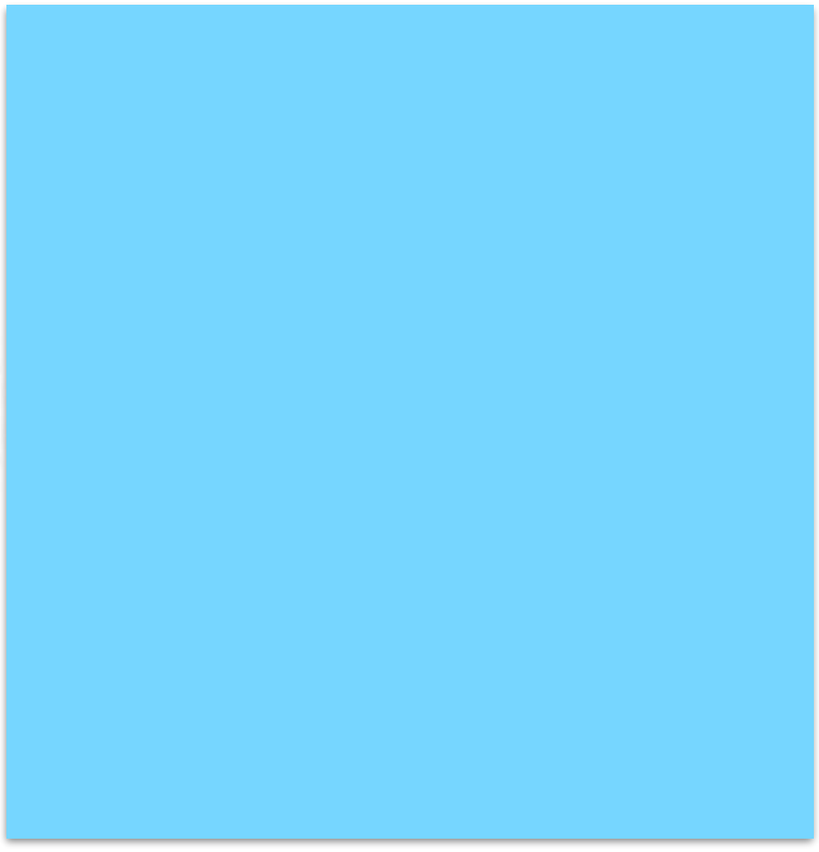




How much is 9 and 5 more?

Features of Early Counting: Counting On

BEWARE OF MAKING COUNTING ON A PROCEDURE:

"PUT THE BIG NUMBER IN YOUR HEAD"

Features of Early Counting: Counting On

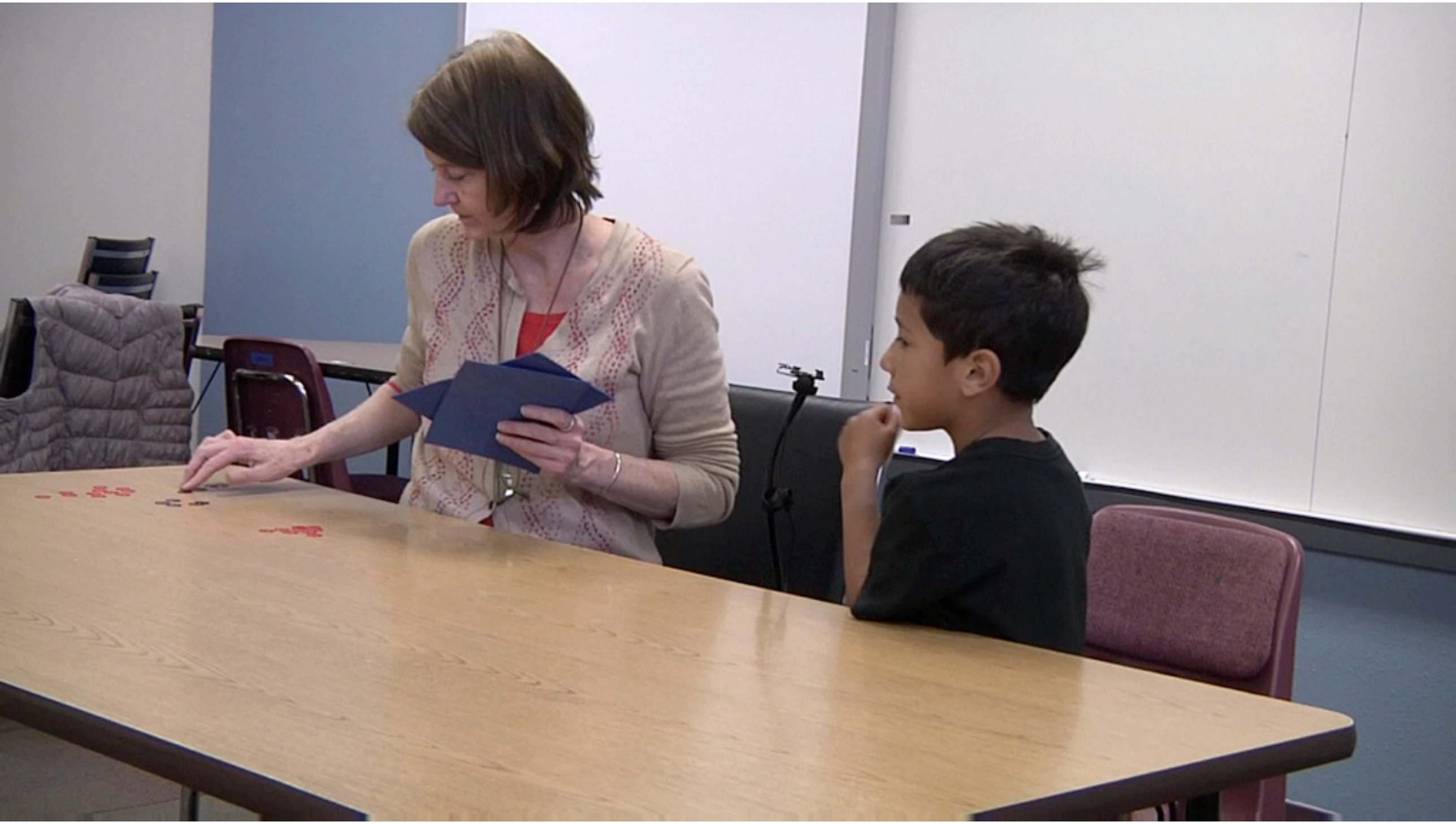
BEWARE OF MAKING COUNTING ON A PROCEDURE:

~~"PUT THE BIG NUMBER IN YOUR HEAD"~~

Stages of Early Counting: Counting On / Counting Back

- ☑ Student understands that, rather than counting everything in the first collection, they can refer to the entire first collection with a single number

Student counts on to add, counts back to subtract



Stages of Early Counting: Where are they in the CCSSM?

- The standards address perceptual counting in detail

- The standards emphasize solving a variety of problem types (combine, separate, compare, missing addend, missing subtrahend)

- Less attention is given to the stages between perceptual counting and solving bare number tasks

- Awareness of the stages is important in making decisions about instruction & support for young students

  What materials will I use? Will I conceal the materials?
  Is it time to begin to connect materials to symbolic notation?

Stages of Early Counting: Kinder CCSSM

Count to tell the number of objects
K.CC.4
Understand the relationship between numbers and quantities; connect counting to cardinality.

a) When counting objects, say the numbers in the standard order, pairing each object with one number and each number with one object.
b) Understand that the last number said tells the number of items counted, the number of items is the same regardless of arrangement or order in which they are counted.

K.CC.5
Count to answer "how many?" questions about up to 20 things; given a number from 1–20, count out that many objects.

Understand addition as putting together & adding to
Understand subtraction as taking apart & taking from
K.OA.1
Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
K.OA.2
Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

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Perceptual Counting

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figurative counting?

Perceptual Counting

Stages of Early Counting: Mathematizing in Kinder

- ☑ Allow time for students to become robust perceptual counters

- ☑ Count collections that are becoming smaller (remember to practice backwards counting sequences)

- ☑ Consider how to advance student thinking as students become robust perceptual counters

- ☑ Introduce the idea of counting collections that are covered up

Stages of Early Counting:
1st Grade CCSSM

Represent and solve problems involving addition and subtraction.
1.OA.1
Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

1.OA.2
Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

Add & subtract within 20
1.OA.5
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

Stages of Early Counting:
1st Grade CCSSM

Represent and solve problems involving addition and subtraction.

1.OA.1

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

1.OA.2

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

(Understand and apply properties of operations and the relationship between addition and subtraction.)

Add & subtract within 20

1.OA.5

Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

Stages of Early Counting:
1st Grade CCSSM

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(Understand and apply properties of operations and the relationship between addition and subtraction.)

Add & subtract within 20

1.OA.5

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(Work with addition and subtraction **equations**.)

Stages of Early Counting: Mathematizing in 1st Grade

- Consider how to advance student thinking:
  How will figurative students develop a counting on strategy?
  How will your students make sense of symbolic notation (equations)?

- Present two collections that are covered up: How many are there in all?

- Put a small number of counters in the second collection (students are more likely to realize they can "count on" when there are only one or two more counters to add)

- Once students develop a counting on strategy to add collections, begin to present tasks that involve removing counters (subtraction)

- Introduce symbolic notation once students have robust counting on / counting back strategies to solve addition & subtractions tasks with counters

Thank you!

This webinar recording and handouts are available at the MC² and New Mexico K-3 PLUS websites.

Please submit any questions or comments to mc2@nmsu.edu.