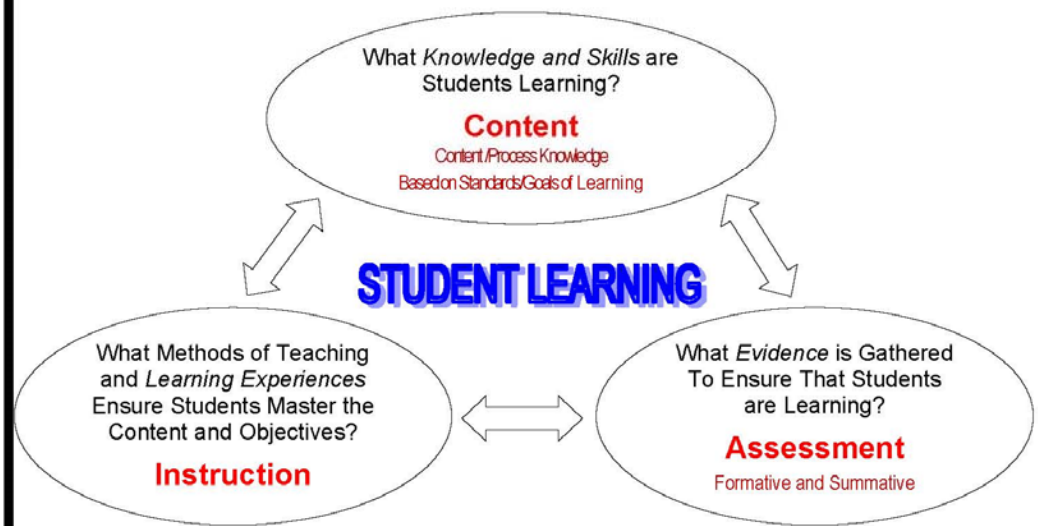
Handouts for MC² Webinar: Exponential Functions, April 2016

THREE GUIDING QUESTIONS IN A STANDARDS-BASED LEARNING ENVIRONMENT

Adapted from Keeping the Focus on Learning: McRel Report 2002



STANDARDS-BASED LEARNING ENVIRONMENT (SBLE) INDICATORS

1. The enacted lesson provided opportunities for students to make conjectures about mathematical ideas.
2. The enacted lesson fostered the development of conceptual understanding.
3. Students explained their responses or solution strategies.
4. Multiple perspectives/strategies were encouraged and valued.
5. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.

Tarr, J.E., Reys, R.E., Reys, B.J. & Chávez. O. (2008). "The Impact of Middle-Grades Mathematics Curricula and the Classroom Learning Environment on Student Achievement." *Journal for Research in Mathematics Education*, Vol. 39, No. 3, pp. 247-280.

How Does an Infection Spread?

One day a couple students came to school, coughing and feverish; they had the flu. Then, within a week or so, it seemed like everyone at the school was getting sick.

Experiment:

1. **Setup:**
    - Place 500 Skittles in the pizza box, with only 2 of the Skittles being RED, and the remaining 498 being other colors.
    - Make a table:

Shake Number	Number of RED Skittles	Number of "other color" Skittles
0	2	498
1		
2		
3		
…		

2. **Procedure:**
    - Shake the pizza box with the 500 Skittles.
    - Count the number of RED Skittles that have an S showing.
    - Remove as many "other color" Skittles as the number of RED Skittles with an S showing.
    - Replace them with RED Skittles.
    - Count the new totals for RED Skittles and "other color" Skittles and record in the table (Hints: you never take RED Skittles out and you should always have 500 Skittles).
    - Repeat the experiment ten more times.

3. **Prediction:**
    - Use a different **color pencil** and make a **prediction** for the next line in your table.
    - Perform one more shake, and also record the actual data. Compare this number with your prediction. Was your prediction accurate? Why or why not?

Data

Shake Number	Number of Red Skittles	Number of other colored Skittles
0	2	498
1	4	496
2	5	495
3	8	492
4	10	490
5	16	484
6	23	477
7	34	466
8	48	452
9	69	431
10	99	401

Analysis:

1. In the context of the spread of infection among the students at the school:
    a. What do the 2 initial RED Skittles represent?

    b. What do the 498 other-colored Skittles represent?

    c. After any shake, what do the RED Skittles represent?

    d. After any shake, what do the other-colored Skittles represent?

    e. After any shake, what do the RED Skittles with an S represent?

    f. After any shake, what do the other-colored Skittles with an S represent?

2. Create a graph that represents the relationship between the Shake Number and the Number of RED Skittles.

3. Write an equation to model the relationship between the Shake Number and the Number of RED Skittles.

Part 1: Understanding the math of Ebola data.

1. There were 10 cases of Ebola reported in January 2014. If the number of Ebola cases increased by 30 each month and continues to increase in this way, then

   a. Create a table of values showing the number of months from January 2014 and the number of Ebola cases. The start of your table of values is:

Month (2014)	Month from January 2014	Number of Ebola cases
January	0	10
February	1	40
…		

   b. Create a graph that describes the relationship between the predicted total number of Ebola cases and the number of months elapsed from January 2014.

   c. Write an equation that describes the relationship between the predicted total number of Ebola cases and the number of months elapsed from January 2014.

   d. If this trend continues, predict the number of Ebola cases by Christmas 2015, Christmas 2016, and Christmas 2017.

Part 2: Understanding the math of Ebola data.

If, instead, the number of Ebola cases increased in such a way that each month the number of cases doubled from the number of cases the previous month and continues to increase in this way, then

a. Create a table of values for the year 2014 showing the number of months from January 2014 and the number of Ebola cases. The start of your table of values might look like the following:

Month (2014)	Month from January 2014	Number of Ebola cases
January	0	10
February	1	20
…		

b. Create a graph that describes the relationship between the predicted total number of Ebola cases and the number of months elapsed from January 2014. Compare this graph, describing similarities and differences, with your graph in task #1.

c. Write an equation that describes the relationship between the predicted total number of Ebola cases and the number of months elapsed from January 2014.

d. If this trend continues, predict the number of Ebola cases by Christmas 2015, Christmas 2016, and Christmas 2017.

Part 3: Understanding the math of Ebola data.

If, instead, the number of Ebola cases increased in such a way that each month the number of cases tripled from the number of cases the previous month and continues to increase in this way, then

a. Create a table of values for the year 2014 showing the number of months from January 2014 and the number of Ebola cases. The start of your table of values might look like the following:

Month (2014)	Month from January 2014	Number of Ebola cases
January	0	10
February	1	30
…		

b. Create a graph that describes the relationship between the predicted total number of Ebola cases and the number of months elapsed from January 2014. Compare this graph, describing similarities and differences, with your graph in task #2. To make this comparison you may want to create and scale a set of axes that can be used to graph the relationship from Task #2 and the relationship from Task #3.

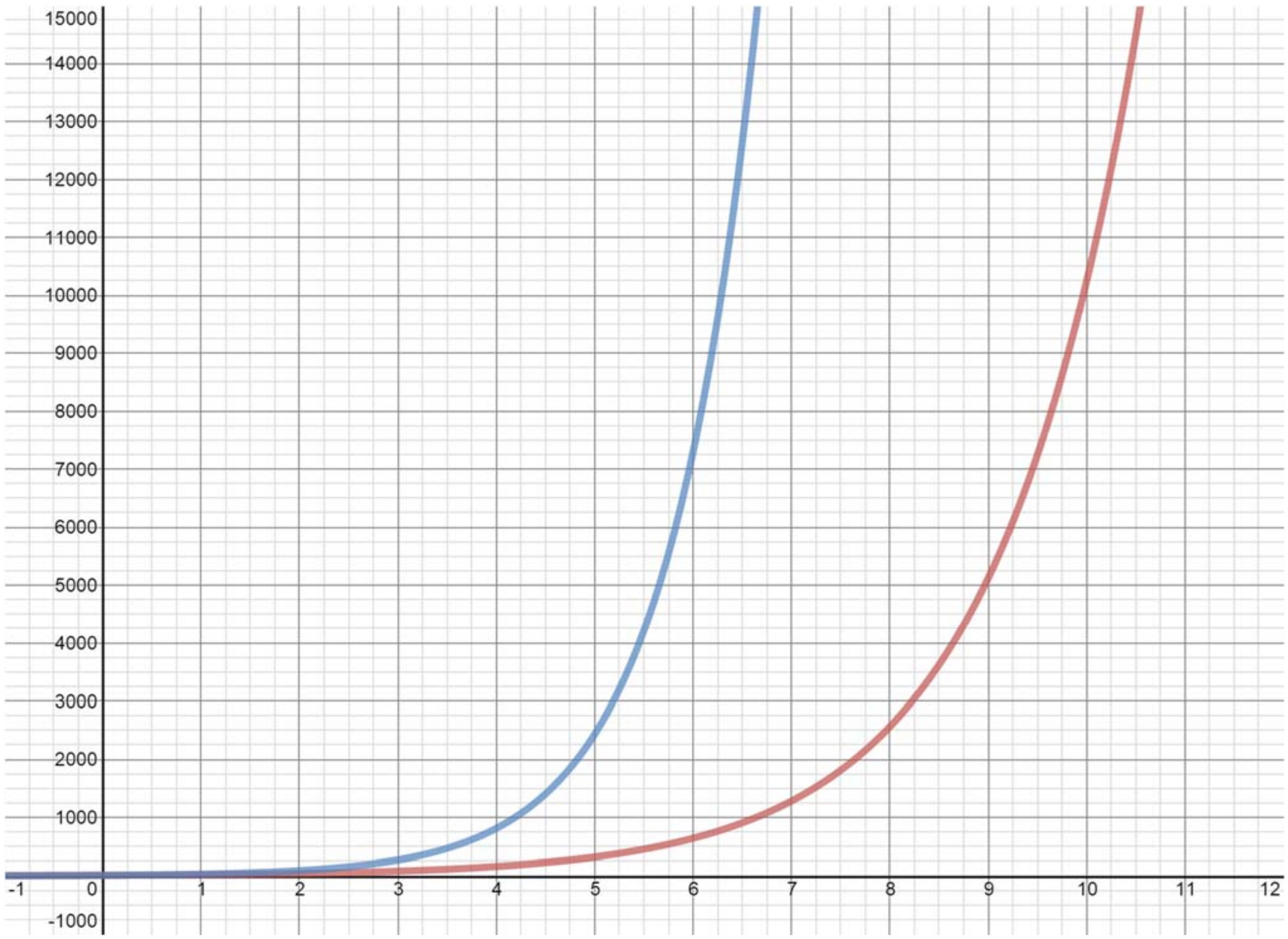
c. Write an equation that describes the relationship between the predicted total number of Ebola cases and the number of months elapsed from January 2014.

d. If this trend continues, predict the number of Ebola cases by Christmas 2015, Christmas 2016, and Christmas 2017.

1. *Time* magazine (10 November 2014) gives the following data for the Number of Ebola cases and the number of deaths from Ebola.

Month (2014)	Number of Ebola cases	Number of Ebola deaths
January	10	10
February	39	34
March	130	82
April	238	158
May	420	224
June	759	467
July	1440	826
August	3417	1818
September	7492	3439
October	13703	4922

Ebola Data


15000
14000
13000
12000
11000
10000
9000
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4000
3000
2000
1000
-1000
-1
0
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11
12

A-SSE **Write expressions in equivalent forms to solve problems**

3. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

A-CED **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-REI **Represent and solve equations and inequalities graphically**

10. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

FUNCTIONS

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate.

Overview

Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems

F-IF Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

   E) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay*.

F-BF Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.★

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F-LE Linear, Quadratic, and exponential models★

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are Numbers and the base b is 2,10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

S-ID Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*