

Geometry Research Lesson Final Report

Introduction

The research lesson topic is the Law of Sines. This lesson's purpose is to engage the students in the learning process by presenting a problem that cannot be solved with the right triangle trigonometry methods they have previously learned. The expected lesson result is that the students will be able to use the Law of Sines to solve a triangle that does not have a right angle. It is also intended that they will see that trigonometry is not such a formidable discipline and that they can easily solve the problems with this new formula. This realization will help to encourage them to be self-confident risk takers.

The lesson is a modification of a lesson previously taught by two of the teachers on the team. The team started with the lesson as it had been taught and then built on its strengths and weaknesses using the MathStar guidelines. The team met before school and during lunch to collaborate on how the lesson would be taught and working on the areas with which we believed the students would have difficulties.

The focus of the lesson is to solve a general triangle - one without a right angle. Students often have a difficult time determining which angle corresponds to each side. They must also understand how to use proportions to solve equations and they must understand how to use their calculators to find the sine of an angle or the inverse sine of a value. The lesson was designed to help students confront the challenge of using the Law of Sines to solve a general triangle. Although the process is relatively straightforward, students often give up because the process is also lengthy. By presenting the topic in the manner chosen by the team, the teacher is able to engage the entire class in the new concept. Because students had not seen this concept before, problems worked and problems assigned were carefully chosen to avoid the ambiguous case which sometimes occurs in Law of Sines problems. It is expected that students will have a sufficient

amount of fundamental knowledge from this lesson to apply to the more complex problems in an Algebra II course.

Student Population

The student population at our high school consists of 68% Anglos, 23% Hispanics, 3.9% Asians, 2.6% Native Americans, and 2% African-Americans. The geometry class in which the lesson study was conducted is a regular geometry class in which the majority of students are sophomores. In a geometry class, students often struggle with visualizing the concepts and applying the concepts to other geometric and algebraic problems. In the math class observed the students are accustomed to working in groups and individually. Most of the students are fairly motivated and are good workers, although there are a few exceptions.

Mathematics Learning

Prior to this lesson the students had learned how to solve proportions and they had learned the basic concepts of right triangle trigonometry. They had solved right triangles using trigonometry. At the beginning of the lesson, students were asked to solve a triangle which did not have a right angle. Although students sometimes attempt to use right triangle trigonometry at this point, none of the students in the class observed did so. This may be attributed to a problem from the test over right triangle trigonometry. In this problem the triangle was not a right triangle. Students had to split it into two triangles with an altitude to one of the bases and use special right triangles to answer the question. Because students had been so recently exposed to this problem, they were very ready to observe that the problems in the new lesson could not be solved by the old methods and none of the students in the class that was observed made the mistake of assuming the triangle was a right triangle. They were not able to solve the triangle, but they knew this immediately and were able to tell why they could not solve it. The students were very adept

at choosing which side was opposite which angle because an innovative approach to right triangle trigonometry had introduced that topic. This was very helpful. The team used all of these previously learned concepts to develop the research lesson. Students will build on their previously acquired knowledge to learn how to solve triangles which do not have a right angle. From this lesson students will move on to solving triangles using the Law of Cosines and then to determining when to use the Law of Sines and when to use the Law of Cosines.

Student Misconceptions

Students often have difficulty determining which angle corresponds to a particular side. Although this particular class did very well with this concept, the teacher needs to be aware that this problem may arise. The teacher needs to stress throughout the lesson and in the lessons to follow that the special angles are only congruent or supplementary if the lines intersected by the transversal are parallel.

Common questions that arise during this lesson are:

- Can we use the Law of Sines to solve a right triangle?
 - Answer: Yes, but it would be simpler to use right triangle trigonometry.
- Will this work on any triangle?
 - Answer: No. It will only work when the information given is ASA (angle-side-angle), AAS (angle-angle-side), or SSA (side-side-angle)

Instructional Strategies

In this lesson, the teacher's introduction to the problem and class discussion are crucial. Working through a problem with the class is a critical component to the learning process. The teacher must emphasize the convention of naming sides and angles in a triangle whereby the side opposite a particular angle is designated by the lower case of the letter used to label the angle. That is, side a is opposite angle A . Because of the

complexity of the problem, it is essential that the students be required to work through the first problem at their desks as the teacher works it on the overhead projector. They must go through the process of obtaining the sine of an angle by using their calculators. With subsequent problems, students are asked to work the problem independently and then to check their answer with the teacher's answer on the overhead.

At the end of the day's lesson, students were again asked to solve the triangle they had been given at the beginning of class. Most of the students solved the problem successfully. The key instructional strategy of this lesson was the use of the familiar concepts learned in the unit on right triangle trigonometry and in the unit on proportionality to extend to the new concept. The team then developed the methods with which we taught the lesson concepts. The extensions of the lesson were developed and strengthened through our discussions.

Changes

The team decided not to make any changes in the research lesson.

Conclusions

Based on observations during the lesson and in the lessons that followed the research lesson, the team learned how truly valuable the approach used was in building students' self confidence in solving complex problems. A similar lesson was presented on the Law of Cosines. Students' scores on the test over these two topics were excellent. Many students who had never before scored above 80% made a perfect score. The lesson study process allowed our team to develop and share the approach that was ultimately so successful with the classes of every member of the team. We were able to work through the discovery process of the lesson as a team. We were also able to discover more extensions to the lesson through team observations.

Engaging Student Interest

This was the students' first introduction to the solution of general triangles using the Law of Sines or any other method. They had recently enjoyed some success in solving right triangles and felt optimistic about their ability to proceed to this new lesson. The students were kept engaged in the lesson by working through the examples at their desks while the teacher worked the examples on the overhead projector. Prompting students with appropriate questions facilitated class communication. The students' understanding of the lesson was assessed through their responses to the questions during class discussion. Closure to the lesson came through having the students solve the problem presented at the beginning of class. They then began the assignment, working in groups and supporting each other in their solutions of the problems. Most students were able to solve the pre-test/post-test problem at the end of the lesson. Others gained additional experience and solved problems successfully in their small groups. Furthermore, at the end of the unit, the results on the test were excellent, with virtually every student receiving a satisfactory grade on the examination, and almost half the class solving every problem on the test correctly. Other teachers on the team experienced similar results when they presented the lesson and when they assessed the students' learning.

Team Learning

The team determined, through this research lesson, that relating this lesson to the previous topic was very successful. Students clearly understood which angles of the triangle were opposite which sides. This was essential to the success of this lesson. Presenting detailed solutions in the examples was also recognized as an important aspect of the success of the lesson.

Lesson Study Process

Impact on Teachers

The Lesson Study process allowed our team to learn how to work with different learning styles and with different ways of problem solving. All of the teachers on the team brought with them prior knowledge of material and of student misconceptions. Without the communication between team members, our discoveries and changes would not have been as meaningful or as many. The lesson may have been good with one or two teachers working on it, but with a team of teachers working at it, the lesson was a great success.

Strengths and Weaknesses

The strengths of the Lesson Study process are:

- Teachers learn with teachers
- Enables teachers to be better risk takers
- Support from facilitators and teachers

The weaknesses of the Lesson Study process are:

- Timetable not known to participants well in advance
- Yearly requirements not clearly defined
- Time out of classroom required
- Guidelines for presentation and paper not clear
- Meetings not well organized/boring

Adapting Lesson Study into School Context

- Professional development time allotted each month
 - Our school has professional development on Thursday mornings. Being allowed to work on the Lesson Study process during this time would be helpful.
- Common prep period

Geometry Research Lesson

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