

# 9-12 MATHEMATICS

**Strand: ALGEBRA, FUNCTIONS, AND GRAPHS**

**Standard:** Students will understand algebraic concepts and applications.

**9-12 Benchmark:** Represent and analyze mathematical situations and structures using algebraic symbols.

Grade	Performance Standards
9-12	<p>Classify numbers and members of the following sets:</p> <ul style="list-style-type: none"><li>• natural</li><li>• whole</li><li>• integers</li><li>• rationals</li><li>• irrationals</li></ul> <ol style="list-style-type: none"><li>2. Simplify numerical expressions using the order of operations, including exponents.</li><li>3. Evaluate the numerical value of expressions of one or more variables that are:<ul style="list-style-type: none"><li>• polynomial</li><li>• rational</li><li>• radical</li></ul></li><li>4. Simplify algebraic monomial expressions raised to a power (e.g., <math>[5xy^2]^3</math>) and algebraic binomial (e.g., <math>[5x^2 + y]^2</math>) expressions raised to a power.</li><li>5. Compare and order polynomial expressions by degree.</li><li>6. Represent and analyze relationships using written and verbal expressions, tables, equations, and graphs, and describe the connections among those representations:<ul style="list-style-type: none"><li>• translate from verbal expression to algebraic formulae (e.g., “Set up the equations that represent the data in the following equation: John’s father is 23 years older than John. John is 4 years older than his sister Jane. John’s mother is 3 years younger than John’s father. John’s mother is 9 times as old as Jane. How old are John, Jane, John’s mother, and John’s father?”)</li><li>• given data in a table, construct a function that represents these data (linear only)</li><li>• given a graph, construct a function that represents the graph (linear only)</li></ul></li><li>7. Know, explain, and use equivalent representations for the same real number including: integers decimals percents ratios scientific notation numbers with integer exponents inverses (reciprocal) prime factoring</li><li>8. Simplify algebraic expressions using the distributive property.</li><li>9. Explain and use the concept of absolute value.</li><li>10. Know, explain, and use equivalent representations for algebraic expressions.</li><li>11. Simplify square roots and cube roots with monomial radicands that are perfect squares</li></ol>

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	<p>or perfect cubes (e.g., <math>9a^2x^4</math>).</p> <p>12. Calculate powers and roots of real numbers, both rational and irrational.</p> <p>13. Solve:</p> <ul style="list-style-type: none"> <li>• formulas for specified variables</li> <li>• radical equations involving one radical</li> </ul> <p>Factor polynomials, difference of squares and perfect square trinomials, and the sum and difference of cubes.</p> <p>Simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.</p> <p>Manipulate simple expressions with + and – exponents.</p> <p>Use the four basic operations (+, -, x, ÷) with:</p> <ul style="list-style-type: none"> <li>• linear expressions</li> <li>• polynomial expressions</li> <li>• rational expressions</li> </ul>
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**Strand: ALGEBRA, FUNCTIONS, AND GRAPHS**  
**Standard:** Students will understand algebraic concepts and applications.

**9-12 Benchmark:** Understand patterns, relations, functions, and graphs.

Grade	Performance Standards
<b>9-12</b>	<ol style="list-style-type: none"> <li>1. Distinguish between the concept of a relation and a function.</li> <li>2. Determine whether a relation defined by a graph, a set of ordered pairs, a table of values, an equation, or a rule is a function.</li> <li>3. Describe the concept of a graph of a function.</li> <li>4. Translate among tabular, symbolic, and graphical representations of functions.</li> <li>5. Explain and use function notation.</li> <li>6. Determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.</li> <li>7. Identify the independent and dependent variables from an application problem (e.g., height of a child).</li> <li>8. Describe the concept of a graph of an equation.</li> <li>9. Understand symmetry of graphs.</li> <li>10. Analyze and describe middle and end (asymptotic) behavior of linear, quadratic, and exponential functions, and sketch the graphs of functions.</li> <li>11. Work with composition of functions (e.g., find f of g when <math>f(x) = 2x - 3</math> and <math>g(x) = 3x - 2</math>), and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.</li> <li>12. Use the quadratic formula and factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.</li> <li>13. Apply quadratic equations to physical phenomena (e.g., the motion of an object under the force of gravity).</li> </ol>

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**Strand: ALGEBRA, FUNCTIONS, AND GRAPHS**

**Standard:** Students will understand algebraic concepts and applications.

**9-12 Benchmark:** Use mathematical models to represent and understand quantitative relationships.

<b>Grade</b>	<b>Performance Standards</b>
9-12	<ol style="list-style-type: none"><li>1. Model real-world phenomena using linear and quadratic equations and linear inequalities (e.g., apply algebraic techniques to solve rate problems, work problems, and percent mixture problems; solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest; apply quadratic equations to model throwing a baseball in the air ).</li><li>2. Use a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools). Express the relationship between two variables using a table with a finite set of values and graph the relationship. Express the relationship between two variables using an equation and a graph:<ul style="list-style-type: none"><li>• graph a linear equation and linear inequality in two variables</li><li>• solve linear inequalities and equations in one variable</li><li>• solve systems of linear equations in two variables and graph the solutions</li><li>• use the graph of a system of equations in two variables to help determine the solution</li></ul>Solve applications involving systems of equations. Evaluate numerical and algebraic absolute value expressions. Create a linear equation from a table of values containing co-linear data. Determine the solution to a system of equations in two variables from a given graph. Generate an algebraic sentence to model real-life situations. Write an equation of the line that passes through two given points.</li></ol> <ol style="list-style-type: none"><li>11. Understand and use:<ul style="list-style-type: none"><li>• such operations as taking the inverse, finding the reciprocal, taking a root, and raising to a fractional power</li><li>• the rules of exponents</li></ul>Verify that a point lies on a line, given an equation of the line, and be able to derive linear equations by using the point-slope formula.</li></ol>

# 9-12 MATHEMATICS

**Strand: ALGEBRA, FUNCTIONS, AND GRAPHS**

**Standard:** Students will understand algebraic concepts and applications.

**9-12 Benchmark:** Analyze changes in various contexts.

Grade	Performance Standards
9-12	<p>Analyze the effects of parameter changes on these functions:</p> <ul style="list-style-type: none"><li>linear (e.g., changes in slope or coefficients)</li><li>quadratic (e.g., <math>f[x-a]</math> changes coefficients and constants)</li><li>exponential (e.g., changes caused by increasing <math>x[x + c]</math> or <math>[a^x]</math>)</li><li>polynomial (e.g., changes caused by positive or negative values of <math>a</math>, or in a constant <math>c</math>)</li></ul> <p>Solve routine two- and three-step problems relating to change using concepts such as:</p> <ul style="list-style-type: none"><li>exponents</li><li>factoring</li><li>ratio</li><li>proportion</li><li>average</li><li>percent</li></ul> <p>Calculate the percentage of increase and decrease of a quantity.</p> <p>Analyze the general shape of polynomial expressions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials).</p> <p>Estimate the rate of change of a function or equation by finding the slope between two points on the graph.</p> <p>Evaluate the estimated rate of change in the context of the problem.</p> <p>Know Pascal's triangle and use it to expand binomial expressions that are raised to positive integer powers.</p>

# 9-12 MATHEMATICS

**Strand: GEOMETRY AND TRIGONOMETRY**

**Standard:** Students will understand geometric concepts and applications.

**9-12 Benchmark:** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Grade	Performance Standards
9-12	<p>Interpret and draw two-dimensional objects and find the area and perimeter of basic figures (e.g., rectangles, circles, triangles, other polygons [e.g., rhombi, parallelograms, trapezoids]).</p> <p>Find the area and perimeter of a geometric figure composed of a combination of two or more rectangles, triangles, and/or semicircles with just edges in common.</p> <p>Find and use measures of sides and interior and exterior angles of triangles and polygons to classify figures (e.g., scalene, isosceles, and equilateral triangles; rectangles [square and non-square]; other convex polygons).</p> <p>Interpret and draw three-dimensional objects and find the surface area and volume of basic figures (e.g., spheres, rectangular solids, prisms, polygonal cones), and calculate the surface areas and volumes of these figures as well as figures constructed from unions of rectangular solids and prisms with faces in common, given the formulas for these figures.</p> <p>Demonstrate an understanding of simple aspects of a logical argument:</p> <ul style="list-style-type: none"><li>• identify the hypothesis and conclusion in logical deduction</li><li>• use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion</li></ul> <p>Demonstrate an understanding of inductive and deductive reasoning, explain the difference between inductive and deductive reasoning, and identify and provide examples of each:</p> <ul style="list-style-type: none"><li>• for inductive reasoning, demonstrate understanding that showing a statement is true for a finite number of examples does not show it is true for all cases unless the cases verified are all cases</li><li>• for deductive reasoning, prove simple theorems</li></ul> <p>Write geometric proofs (including proofs by contradiction), including:</p> <ul style="list-style-type: none"><li>• theorems involving the properties of parallel lines cut by a transversal line and the properties of quadrilaterals</li><li>• theorems involving complementary, supplementary, and congruent angles</li><li>• theorems involving congruence and similarity</li><li>• the Pythagorean theorem (tangram proof)</li></ul>

# 9-12 MATHEMATICS

**Strand: GEOMETRY AND TRIGONOMETRY**

**Standard:** Students will understand geometric concepts and applications.

**9-12 Benchmark:** Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Grade	Performance Standards
9-12	<ol style="list-style-type: none"><li>1. Demonstrate understanding of the construction of the coordinate plane, know the names of the origin, coordinate axes and four quadrants, draw and label them correctly, find the coordinates of an indicated point, and plot a point with given coordinates.</li><li>2. Determine the midpoint and distance between two points within a coordinate system and relate these ideas to geometric figures in the plane (e.g., find the center of a circle given two endpoints of a diameter of the circle).</li><li>3. Given two linear equations, determine whether the lines are parallel, perpendicular, or coincide.</li><li>4. Use basic geometric ideas (e.g., the Pythagorean theorem, area, and perimeter of objects) in the context of the Euclidean Plane, calculate the perimeter of a rectangle with integer coordinates and sides parallel to the coordinate axes and with sides not parallel.</li></ol>

**Strand: GEOMETRY AND TRIGONOMETRY**

**Standard:** Students will understand geometric concepts and applications.

**9-12 Benchmark:** Apply transformations and use symmetry to analyze mathematical situations.

Grade	Performance Standards
9-12	<p>Describe the effect of rigid motions on figures in the coordinate plane and space that include rotations, translations, and reflections:</p> <p>determine whether a given pair of figures on a coordinate plane represents the effect of a translation, reflection, rotation, and/or dilation</p> <p>sketch the planar figure that is the result of a given transformation of this type</p> <p>Deduce properties of figures using transformations that include translations, rotations, reflections, and dilations in a coordinate system:</p> <ul style="list-style-type: none"><li>• identify congruency and similarity in terms of transformations</li><li>• determine the effects of the above transformations on linear and area measurements of the original planar figure</li></ul>

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**Strand: GEOMETRY AND TRIGONOMETRY**

**Standard:** Students will understand geometric concepts and applications.

**9-12 Benchmark:** Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grade	Performance Standards
9-12	<p>Solve real-world problems using congruence and similarity relationships of triangles (e.g., find the height of a pole given the length of its shadow).</p> <p>Solve problems involving complementary, supplementary, and congruent angles.</p> <p>Solve problems involving the perimeter, circumference, area, volume, and surface area of common geometric figures (e.g., “Determine the surface area of a can of height <math>h</math> and radius <math>r</math>. How does the surface area change when the height is changed to <math>3h</math>? How does the surface area change when the radius is changed to <math>3r</math>? How does the surface area change when both <math>h</math> and <math>r</math> are doubled?”).</p> <p>Solve problems using the Pythagorean theorem (e.g., “Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder”).</p> <p>Understand and use elementary relationships of basic trigonometric functions defined by the angles of a right triangle (e.g., “What is the radius of a circle with an inscribed regular octagon with the length of each side being exactly 2 feet?”).</p> <p>Use trigonometric functions to solve for the length of the second leg of a right triangle given the angles and the length of the first leg. (e.g., “A surveyor determines that the angle subtended by a two-foot stick at right angles to his transit is exactly one degree. What is the distance from the transit to the base of the measuring stick?”).</p> <p>Know and use angle and side relationships in problems with special right triangles (e.g., 30-, 45-, 60-, and 90-degree triangles).</p>

# 9-12 MATHEMATICS

**Strand: DATA ANALYSIS AND PROBABILITY**

**Standard:** Students will understand how to formulate questions, analyze data, and determine probabilities.

**9-12 Benchmark:** Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Grade	Performance Standards
9-12	<ol style="list-style-type: none"> <li>Understand the differences between the various methods of data collection.</li> <li>Know the characteristics of a well-designed and well-conducted survey:               <ul style="list-style-type: none"> <li>differentiate between sampling and census</li> <li>differentiate between a biased and an unbiased sample</li> </ul> </li> <li>Know the characteristics of a well-designed and well-conducted experiment:               <ul style="list-style-type: none"> <li>differentiate between an experiment and an observational study</li> <li>recognize sources of bias in poorly designed experiments</li> </ul> </li> <li>Understand the role of randomization in well-designed surveys and experiments.</li> </ol>

**Strand: DATA ANALYSIS AND PROBABILITY**

**Standard:** Students will understand how to formulate questions, analyze data, and determine probabilities.

**9-12 Benchmark:** Select and use appropriate statistical methods to analyze data.

Grade	Performance Standards
9-12	<ol style="list-style-type: none"> <li>Understand the meaning of measurement data and categorical data, and of the term “variable.”</li> <li>Understand the meaning of “univariate” (i.e., one variable) and “bivariate” (i.e., two variable) data.</li> <li>For univariate data, be able to display the distribution and describe its shape using appropriate summary statistics, and understand the distinction between a statistic and a parameter:               <ul style="list-style-type: none"> <li>construct and interpret frequency tables, histograms, stem and leaf plots, and box and whisker plots</li> <li>calculate and apply measures of central tendency (mean, median, and mode) and measures of variability (range, quartiles, standard deviation)</li> <li>compare distributions of univariate data using back-to-back stem and leaf plots and parallel box and whisker plots</li> </ul> </li> <li>For bivariate data, be able to display a scatter plot and describe its shape:               <ul style="list-style-type: none"> <li>fit a linear model to a set of data using technological tools</li> <li>describe and interpret the relationship/correlation between two variables using technological tools</li> </ul> </li> </ol>

# 9-12 MATHEMATICS

**Strand: DATA ANALYSIS AND PROBABILITY**

**Standard:** Students will understand how to formulate questions, analyze data, and determine probabilities.

**9-12 Benchmark:** Develop and evaluate inferences and predictions that are based on data.

Grade	Performance Standards
9-12	<ol style="list-style-type: none"><li>1. Compare and draw conclusions between two or more sets of univariate data using basic data analysis techniques and summary statistics.</li><li>2. Draw conclusions concerning the relationships among bivariate data:<ul style="list-style-type: none"><li>• make predictions from a linear pattern in data</li><li>• determine the strength of the relationship between two sets of data by examining the correlation</li><li>• understand that correlation does not imply a cause-and-effect relationship</li></ul></li><li>3. Use simulations to explore the variability of sample statistics from a known population and construct sampling distributions.</li><li>4. Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.</li><li>5. Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions.</li></ol>

**Strand: DATA ANALYSIS AND PROBABILITY**

**Standard:** Students will understand how to formulate questions, analyze data, and determine probabilities.

**9-12 Benchmark:** Understand and apply basic concepts of probability.

Grade	Performance Standards
9-12	<p>Explain the concept of a random variable.</p> <p>Understand the concept of probability as relative frequency.</p> <p>Use simulations to compute the expected value and probabilities of random variables in simple cases.</p> <p>Distinguish between independent and dependent events.</p> <p>Understand how to compute the probability of an event using the basic rules of probability:</p> <ul style="list-style-type: none"><li>• complement rule</li><li>• addition rule (disjoint and joint events)</li><li>• multiplication rule (independent events)</li><li>• conditional probability</li></ul>

# 9-12 MATHEMATICS

## Strand: ALGEBRA, FUNCTIONS, AND GRAPHS Guidance for Further Study

### 9-12 Topics for Further Study

#### Solving equations, inequalities and systems

*(As students encounter ever more sophisticated mathematical situations, they will need to be able to generate and solve a variety of equations, inequalities, and systems. They begin by studying more complex linear and quadratic equations and systems.)* Students will be able to:

- solve three-by-three linear systems
- solve two-by-two linear quadratic and quadratic-quadratic systems
- solve and graph equations and inequalities involving absolute value
- solve quadratic inequalities by factoring

#### Polynomials

*(Students will extend the concept of solving linear equations to higher degree polynomials. These polynomials can be used to more accurately describe real-world phenomena.)* Students will be able to:

- factor polynomials of degree higher than two using the fundamental theorem of algebra (e.g. an  $n$ th degree polynomial has at most  $n$  distinct linear factors), integral and rational zero theorems, and factor and remainder theorems
- perform the four basic operations on complex numbers
- factor polynomials using complex numbers
- graph polynomials using the intermediate value theorem
- graph and interpret the conic sections

#### Functions

*(The language and properties of functions are essential to understanding the components of higher mathematics. Functions are the fundamental objects on which students operate in some higher mathematics and are among the building blocks of higher mathematics.)* Students will be able to:

- find and use inverse functions involving ordered pairs, graphs, and explicit statements of a function rule
- examine and graph piece-wise defined functions, including the use of the properties of continuity and discontinuity
- graph rational functions and locate zeros and horizontal and vertical asymptotes

# 9-12 MATHEMATICS

## Strand: GEOMETRY AND TRIGONOMETRY Guidance for Further Study

### 9-12 Topics for Further Study

#### Logs and exponential functions

*(Logs and exponential functions provide tools for more sophisticated modeling and applications for understanding real-life phenomena. Higher mathematics requires regular and successful use of logs and exponents to move beyond polynomials.)* Students will be able to:

- operate with logs and exponential functions on the basis of their inverse relationship
- identify the concept of  $e$
- use exponential functions and common and natural logs to understand real-life situations (e.g., half-life, amortization, logistic growth)
- use logs and exponents to solve equations

#### Trigonometry concepts

*(Trigonometry allows a student to consider periodic functions.)* Students will be able to:

- graph all six trigonometric functions using radian measure, their domains and ranges, and the exact values of the five angles of the six trigonometric functions
- demonstrate an understanding of trigonometric functions as circular functions using symmetry
- solve trigonometric equations
- verify trigonometric identities
- apply trigonometric functions to solve physical problems, including the use of the laws of sines and cosines

#### Series and sequences

*(As students progress toward higher mathematics, they will need an understanding of sequences and functions whose domains are sets of whole numbers as opposed to sets of real numbers [e.g., discrete functions versus continuous functions]. Infinite geometric series provide one way to begin a discussion about limits.)* Students will be able to:

- use algebraic techniques to generate the specific formulas for arithmetic and geometric sequences and series
- extend the concept of series to infinite geometric series
- use the language and notation of limits
- use mathematical induction to prove various mathematical statements

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